Pure Borrow

Linear Haskell Meets Rust-Style Borrowing

Yusuke Matsushita Kyoto University & MPI-SWS

Sept 15, 2026 @ Bristol PLRG

About Me

- **Yusuke Matsushita　松下 祐介**
  - July 15, 1996, Osaka
- **’15–’24　Bachelor, Master, Ph.D. at UTokyo**
  - **Computer science — Programming languages**
    - Supervised by **Prof. Naoki Kobayashi**
- **’25–　Assist. Prof. at Hakubi, KyotoU**
  - Focus on **Rust**
  - **’26–　Visiting scientist** at **MPI-SWS**
    - **Germany, Prof. Derek Dreyer**

About Me

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Computer Science Programming Languages Machine Learning

TITLE	CITED BY	YEAR
RustHorn: CHC-based Verification for Rust Programs Y Matsushita, T Tsukada, N Kobayashi ACM Transactions on Programming Languages and Systems (TOPLAS) 43 (4), 1-54	109	2021
RustHornBelt: A Semantic Foundation for Functional Verification of Rust Programs with Unsafe Code Y Matsushita, X Denis, JH Jourdan, D Dreyer Proceedings of the 43rd ACM SIGPLAN Conference on Programming Language …	95	2022
RustHorn: CHC-Based Verification for Rust Programs Y Matsushita, T Tsukada, N Kobayashi European Symposium on Programming (ESOP), 484-514	65	2020
Nola: Later-Free Ghost State for Verifying Termination in Iris Y Matsushita, T Tsukada Proceedings of the ACM on Programming Languages 9 (PLDI)	8	2025
Borrowable Fractional Ownership Types for Verification T Nakayama, Y Matsushita, K Sakayori, R Sato, N Kobayashi International Conference on Verification, Model Checking, and Abstract …	7	2024

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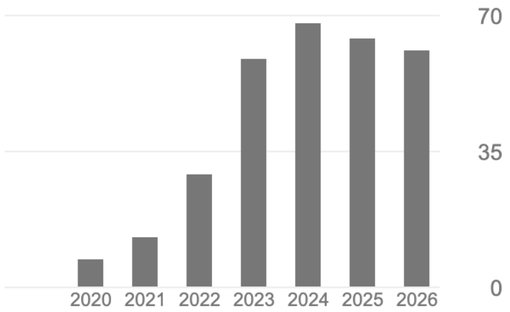

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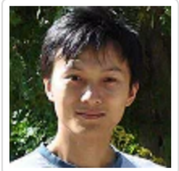

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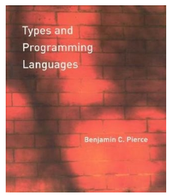
Japan

Up to Now

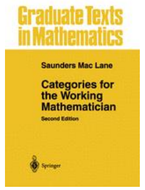
High Schooler — **Undergrad** — **Grad** — **Now**

Haskell

λ-calculus Monad



Type theory



Category theory

PL

Rust

Fast like C/C++,
Safe like Haskell?

Study PL!

RustHorn
ESOP '20 & TOPLAS '21

Rust → Functional

RustHornBelt
PLDI '22

Logic for RustHorn

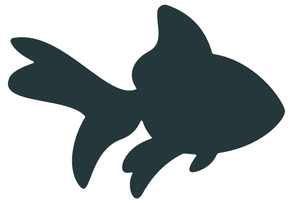
Nola
PLDI '25

Pure Borrow
PLDI '26

Haskell Meets Rust!

This Talk's Focus

Matsushita & Ishii, PLDI '26

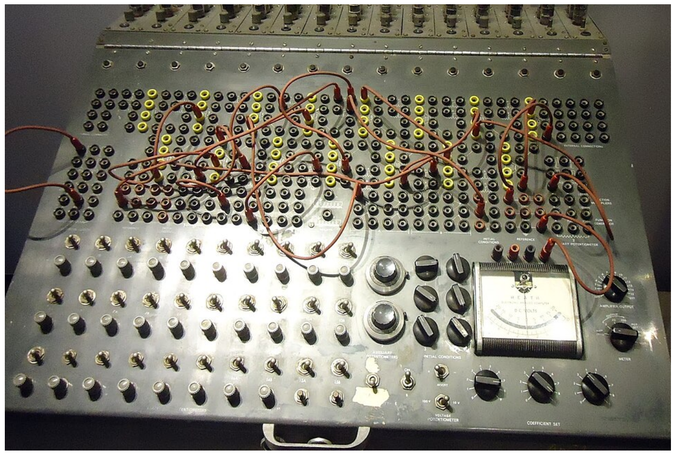
Pure Borrow 

Rust-Style Borrowing in Linear Haskell with Purity

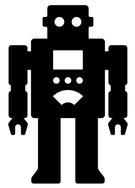
Introduction

Software

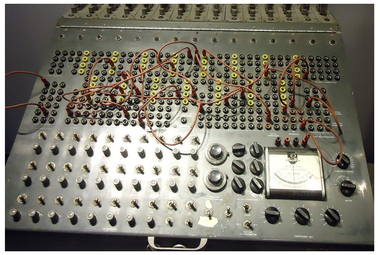
Software



Software embodies human ideals 

on finite physical machines 

Two Major Paradigms of Programming



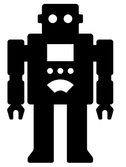
Functional

High-level abstraction

Well-behaved

Safe & Predictable

Imperative



Low-level control

Well-tuned

Performative

Functional, with Purity

Haskell '90

λ-calculus '36

Purity

Same result regardless of timing

Well-behaved, esp. *parallelism*

Purity = Referential Transparency

Super Well-behaved!

Deterministic

Same result across executions

$(t, t) \equiv \textbf{let}\ x = t\ \textbf{in}\ (x, x)$

Order-insensitive

Commutes with other computations

$(t, u) \equiv \textbf{let}\ x = u\ \textbf{in}\ (t, x)$

✔ ***Predictable***

✔ ***Equations***

Parallelism, Lazy evaluation, ...

Reasoning & Optimization

e.g. $\text{map}\ (g \circ f) \equiv \text{map}\ g \circ \text{map}\ f$

Functional, More Imperative

Linear Haskell *Bernardy+ '18*

Mutate! **Linear Types** *'90–*

Performance?

Haskell *'90*

Purity

Same result regardless of timing

λ-calculus *'36*

Well-behaved*, esp.* ***parallelism***

Linear Types

Wadler '90, Bernardy+ '18

- **Original Haskell** **Persistent** data → **Copy**

$$\text{Vec a} \to \text{Int} \to (\text{a} \to \text{a}) \to \text{Vec a}$$

`[0,1,2,…,999]` `0` `(+ 7)` `[7,1,2,…,999]` ***Pure!***

`[0,1,2,…,999]`

- **Linear Haskell** **Linear** → **Mutate** in place!

Linear arrow $\multimap$ *Argument is used exactly* ***once***

$$\underline{\text{Vec a}} \multimap \text{Int} \to (\text{a} \multimap \text{a}) \multimap \underline{\text{Vec a}}$$

Direct ownership passing

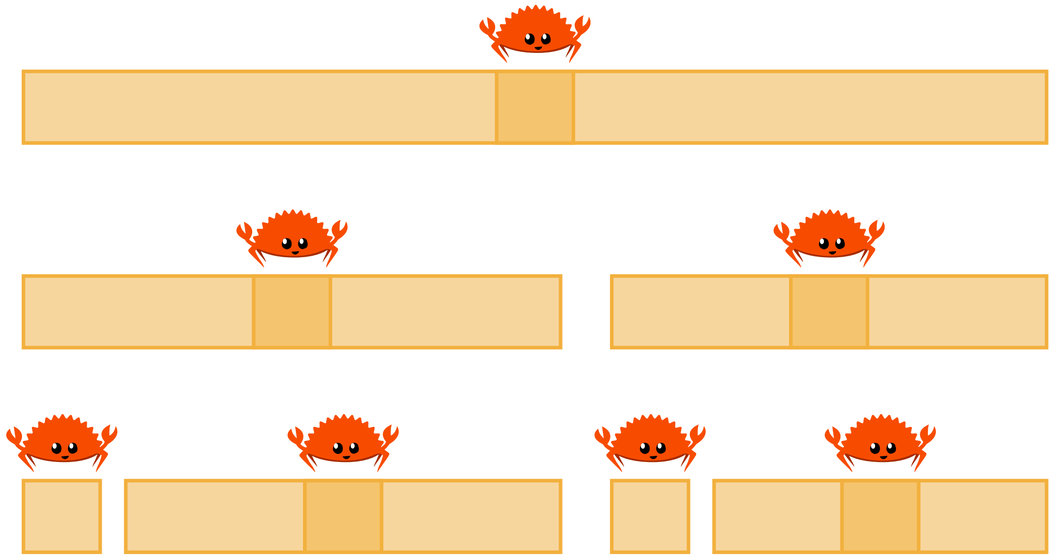
~~`[0,1,2,…,999]`~~ `0` `(+ 7)` `[7,1,2,…,999]` ***Pure!***

No aliases!

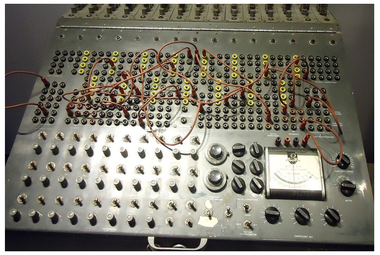
Challenge

Separate mutations to disjoint subregions

E.g., **Parallel quicksort**



Two Major Paradigms of Programming



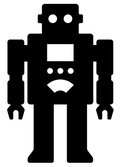
Functional

High-level abstraction

Well-behaved

Safe & Predictable

Imperative

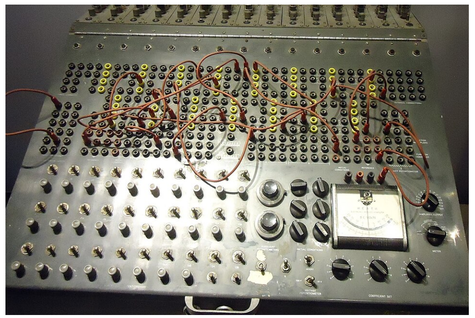


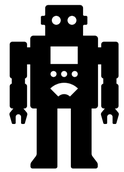
Low-level control

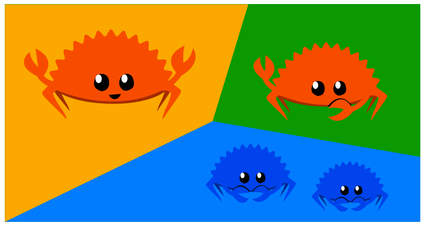
Well-tuned

Performative

Imperative, More Functional







Ownership & Region Types *'94–*

Well-behaved

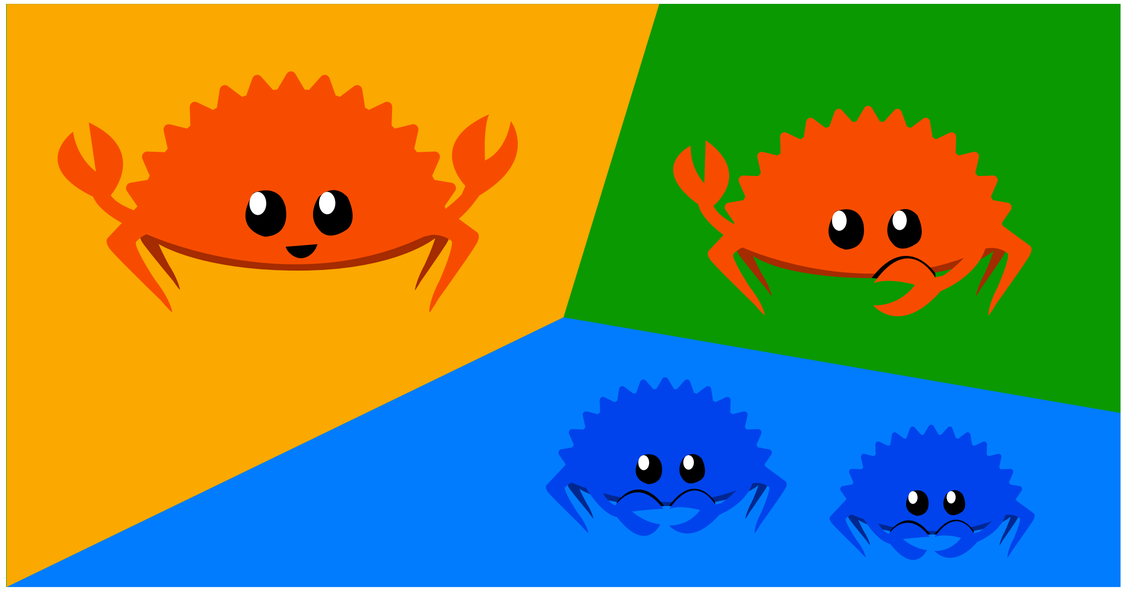
Memory corruption, Race condition, …

'72

'85

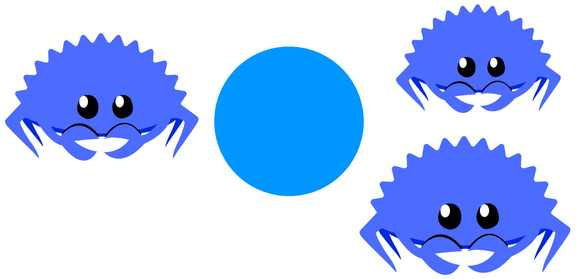
Ownership

Separate Mutable State



XOR

Aliasing



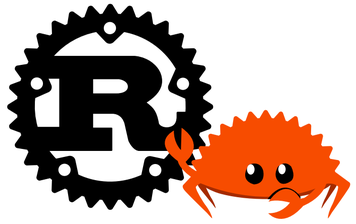
→ No dangerous interference between computations

Imperative, More Functional with Borrows

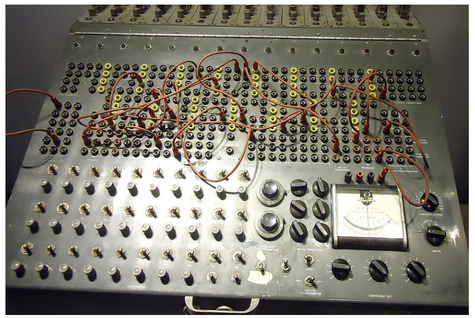
Borrow **Non-local!**

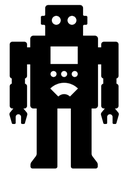
Separate ownership by time

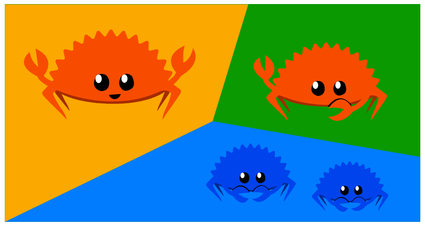
Mutation w/o ownership passing



Rust '15







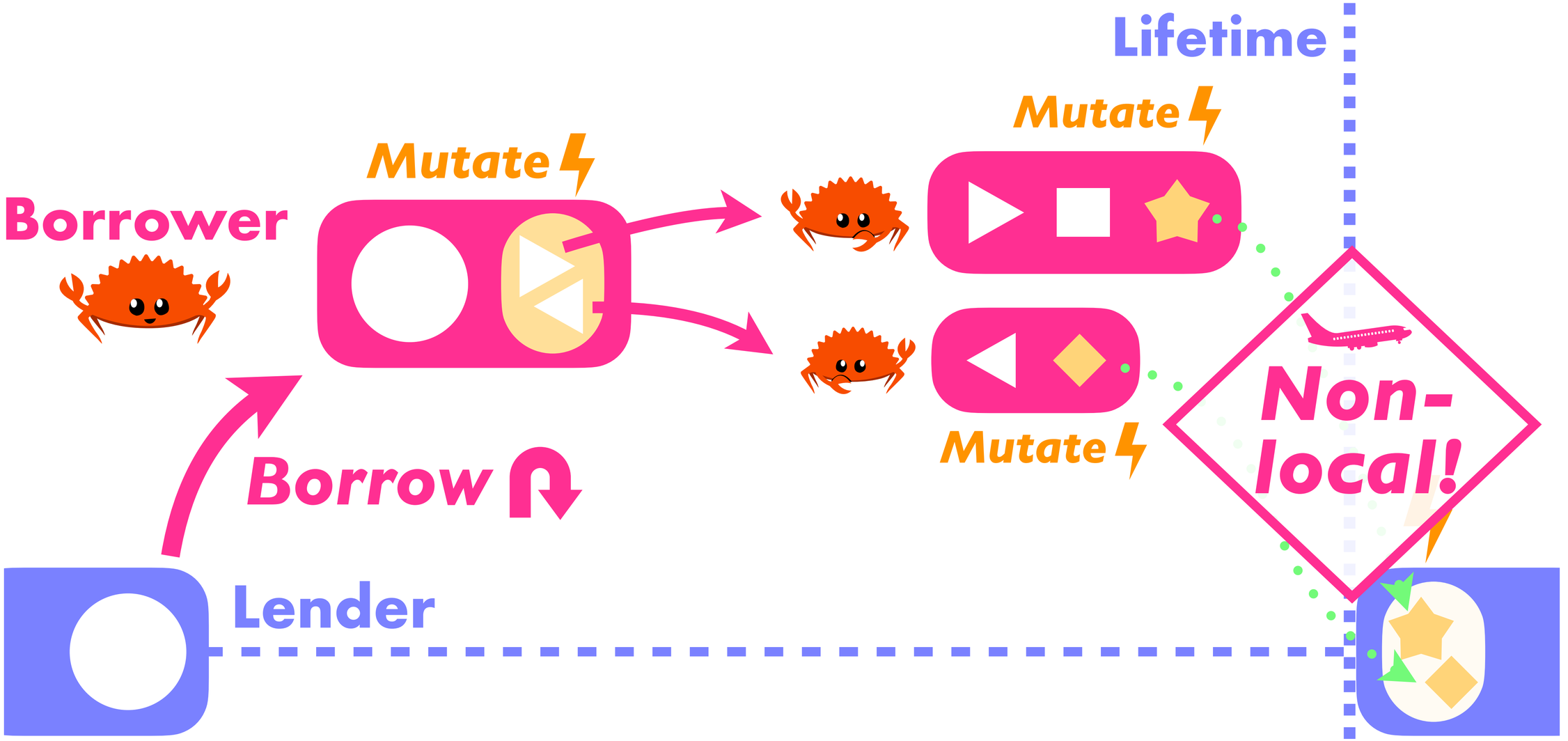
Ownership & Region Types '94–

Well-behaved

Memory corruption, Race condition, …

'72 '85

Rust-style Borrow

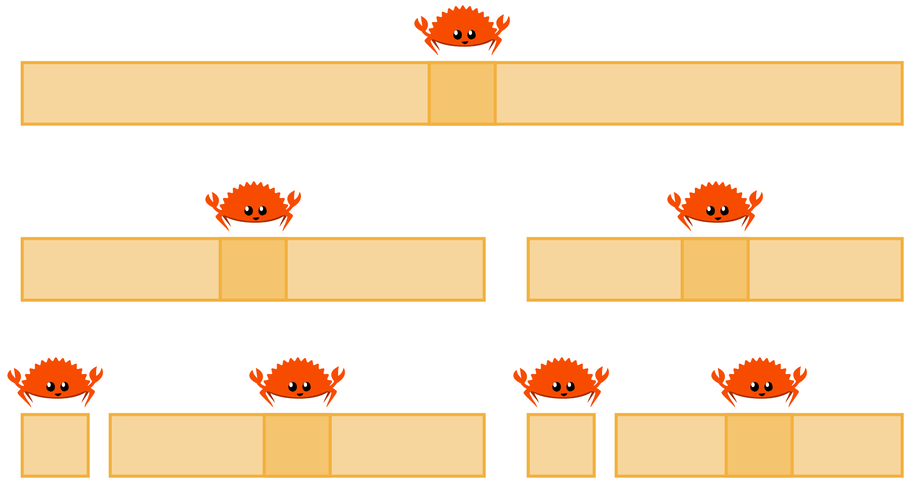


Borrowing Solved! But, ...

✔ Borrow

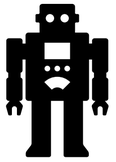
Separate mutations to disjoint subregions

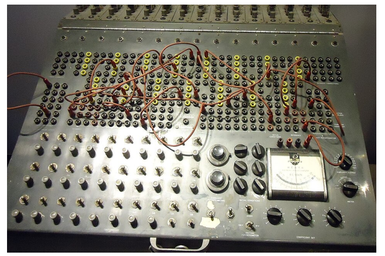
E.g., **Parallel** quicksort



Purity?

Challenge





Purity

Same result regardless of timing

Borrow **Non-local!**

Separate ownership by time

Mutation w/o ownership passing

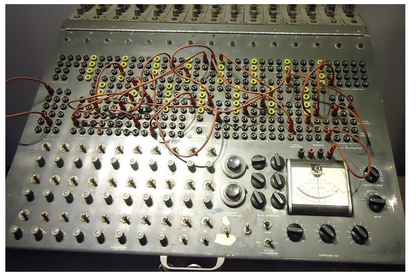
Linear Haskell

Rust

Our Work

Matsushita & Ishii, PLDI '26

Pure Borrow



Rust-Style Borrowing in Linear Haskell with Purity

High-Level Comparison

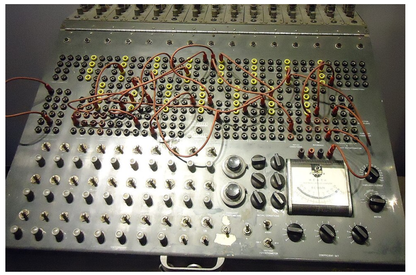
	Pure 	Mutate	Borrow	Parallel	Leak-free
Pure Only	●	○		●	
ST Monad	●	●	○	○	○
Rust	○	●	●	●	○
Basic Linear Haskell	●	●	○	●	●
Pure Borrow	●	●	●	●	●

Our Work

Matsushita & Ishii, PLDI '26

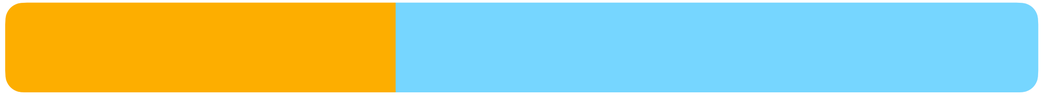
Pure Borrow

Rust-Style Borrowing in Linear Haskell with Purity



- ✦ **Pure & Mutate & Parallelize** *Parallel quicksort*
- ✦ **Just APIs** *Rich features of Haskell*
- ✦ **Metatheory: Why Pure** WIP *Histories for borrows*

This Talk



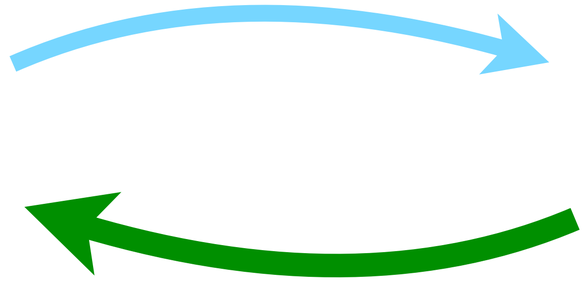
25/66

- ✦ Overview
- ✦ Case Study
- ✦ Metatheory
- ✦ Related Work
- ✦ Conclusion

Overview

Preliminary **Linearity in Linear Haskell**

```
data Ur a where
  Ur :: a → Ur a
```



Unrestricted ω

Default

Linear 1

Opt-in

```
lin :: (Lin ⊸ Ur a) ⊸ Ur a
```

cf. runST :: (∀ s. ST s a) → a

```
class Lin
```

Linearity witness *Spiwack+ '22*

Used arbitrarily finite times *But never in* Ur

Preliminary **Mutable State in Linear Haskell**

Vector **data** Vec a

newVec :: Lin ⊸ [a] ⊸ Vec a

freeVec :: Vec a ⊸ [a]

class Movable a **where**
move :: a ⊸ Ur a

Before Pure Borrow

modifyAt′ :: Vec a ⊸
Int → (a ⊸ a) ⊸ Vec a

readAt′ :: Movable a ⇒
Vec a ⊸ Int → (Ur a, Vec a)

Direct ownership passing!

Preliminary **Example of Basic Linear Haskell**

```
example :: Ur (Int, [Int])
example = lin do°
  v ← newVec [0,1,2]
  v ← modifyAt' v 0 (+ 3); v ← modifyAt' v 2 (+ 5)
  v ← modifyAt' v 0 (× 4)
  (Ur n, v) ← readAt' 0 v
  move (n, freeVec v)
```

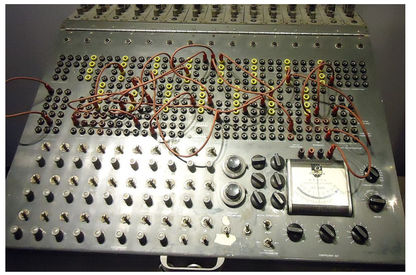
```
do° p ← t; c ≡
  let !p = t in do° c
```

Direct ownership passing!

```
Ur (12, [12, 1, 7])
```

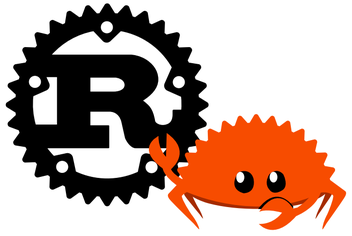
Our Work

Matsushita & Ishii, PLDI '26

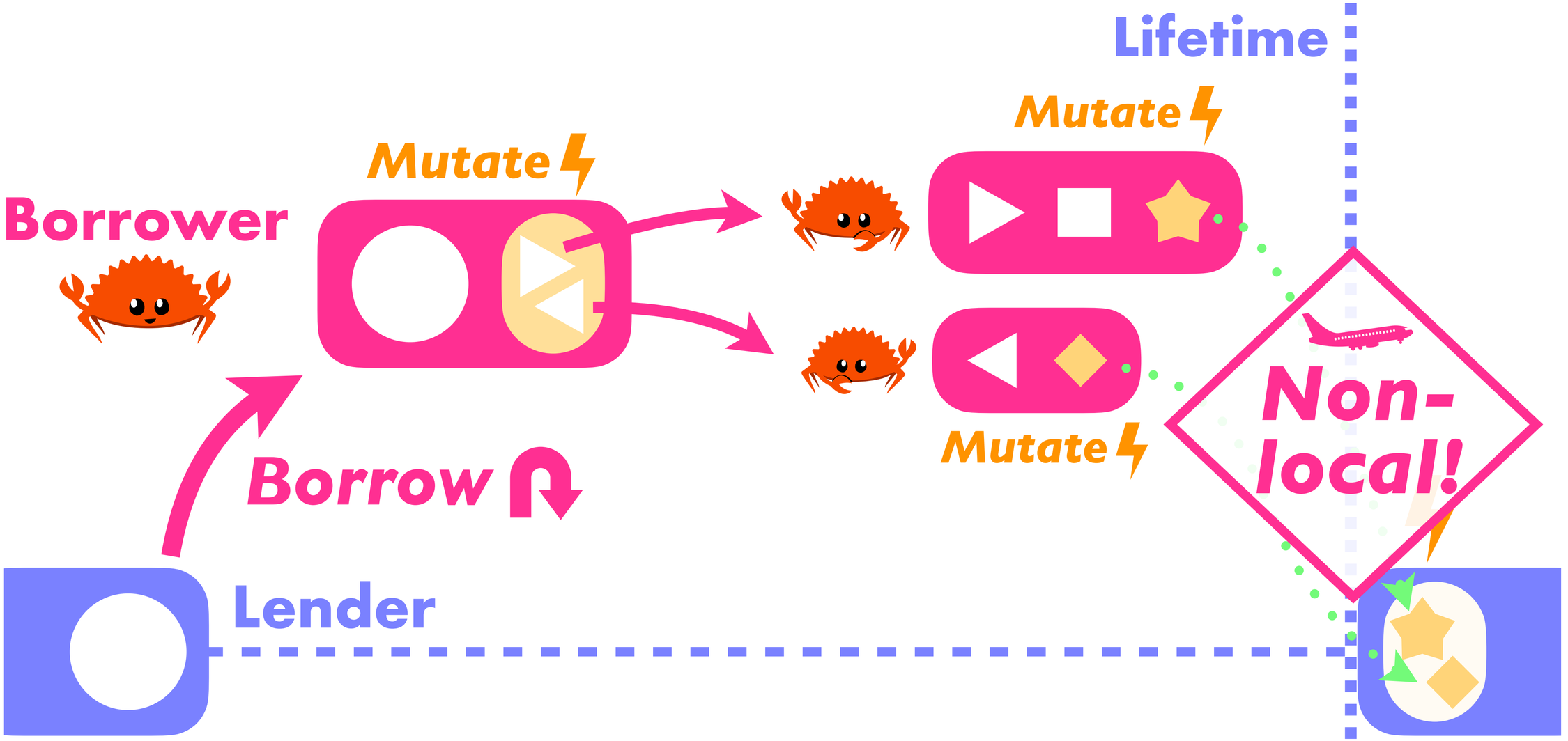


Pure Borrow

Rust-Style Borrowing in Linear Haskell with Purity



Recap **Rust-Style Borrow**

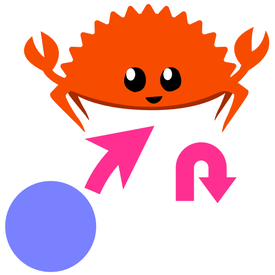


Core APIs

Mutable borrower **Lender**

Borrow $\text{borrow} :: \text{Lin} \multimap a \multimap (\text{Mut}^{\alpha}\ a, \text{Lend}^{\alpha}\ a)$

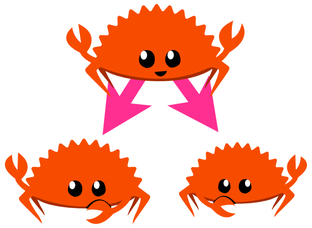
newtype of a



Mutate $\text{modifyAt} :: \text{Mut}^{\alpha}\ (\text{Vec}\ a) \multimap \text{Int} \to (a \multimap a) \multimap \text{BO}^{\alpha}\ (\text{Mut}^{\alpha}\ (\text{Vec}\ a))$

Monad

Split $\text{splitAt} :: \text{Mut}^{\alpha}\ (\text{Vec}\ a) \multimap \text{Int} \to (\text{Mut}^{\alpha}\ (\text{Vec}\ a), \text{Mut}^{\alpha}\ (\text{Vec}\ a))$



α has ended

Reclaim $\text{reclaim} :: \text{End}^{\alpha} \Rightarrow \text{Lend}^{\alpha}\ a \multimap a$

BO Monad

$\mathrm{BO}^{\alpha}\ \mathrm{a}$ ***Computation permitted only before α ends***

Borrower accesses $<$ ***α ends*** $\leq$ ***Lender reclaim***

BO^{α} End^{α}

Pure $\mathrm{runBO} :: \mathrm{Lin} \multimap (\forall\, \alpha.\ \mathrm{BO}^{\alpha}\ (\mathrm{End}^{\alpha} \Rightarrow \mathrm{a})) \multimap \mathrm{a}$

Rank-2 poly

cf. $\mathrm{runST} :: (\forall\, \mathrm{s.}\ \mathrm{ST}^{\mathrm{s}}\ \mathrm{a}) \to \mathrm{a}$

Linearity ensures commutativity

Parallel $\mathrm{parBO} :: \mathrm{BO}^{\alpha}\ \mathrm{a} \multimap \mathrm{BO}^{\alpha}\ \mathrm{b} \multimap \mathrm{BO}^{\alpha}\ (\mathrm{a}, \mathrm{b})$

cf. ST is not parallelizable

Example

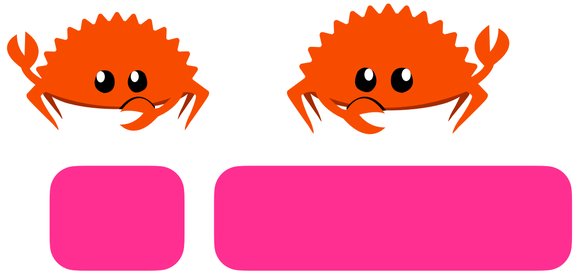
```
example :: Ur [Int]
example = lin do°
  v ← newVec [0, 1, 2]
  runBO do
    let !(mv, lv) = borrow v
    mut_example mv
    pure $ move $
      freeVec $ reclaim lv
                Ur [12, 1, 7]
```

```
mut_example ::
      Mut^α (Vec Int) ⊸ BO^α ()
mut_example mv = do
  mv ← modifyAt mv 0 (+ 3)
  mv ← modifyAt mv 2 (+ 5)
  mv ← modifyAt mv 0 (× 4)
  pure $ consume mv

class Consumable a where
  consume :: a ⊸ ()
```

Example with Parallelization

```
example :: Ur [Int]
example = lin do°
  v ← newVec [0, 1, 2]
  runBO do
    let !(mv, lv) = borrow v
    mut_example mv
    pure $ move $
      freeVec $ reclaim lv
                  Ur [12, 1, 7]
```

```
mut_example mv =
  let !(mv₀, mv₁) = splitAt mv 1 in
  consume <$> parBO
    (do mv₀ ← modifyAt mv₀ 0 (+ 3)
        modifyAt mv₀ 0 (× 4))
    (modifyAt mv₁ 1 (+ 5))
```

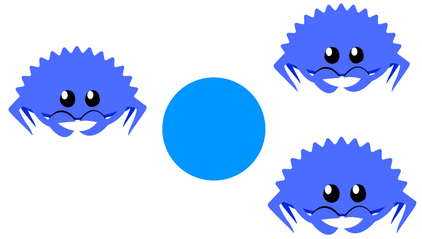


Parallelize!

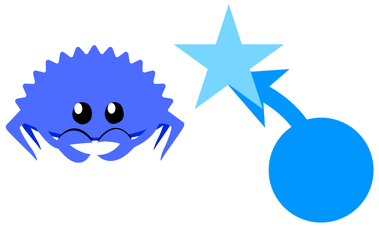
Share

Shared borrower

Share share :: Mut^{α} a ⊸ Ur (Share^{α} a)

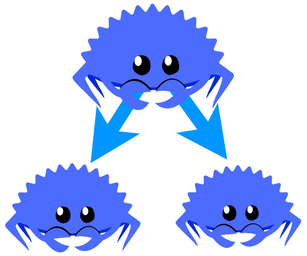


Read copyAt :: Copyable a ⇒
Share^{α} (Vec a) → Int → BO^{α} (Ur a)



class Copyable a **where**
copy :: Share^{α} a → a

Split splitAt :: Share^{α} (Vec a) ⊸
Int → (Share^{α} (Vec a), Share^{α} (Vec a))



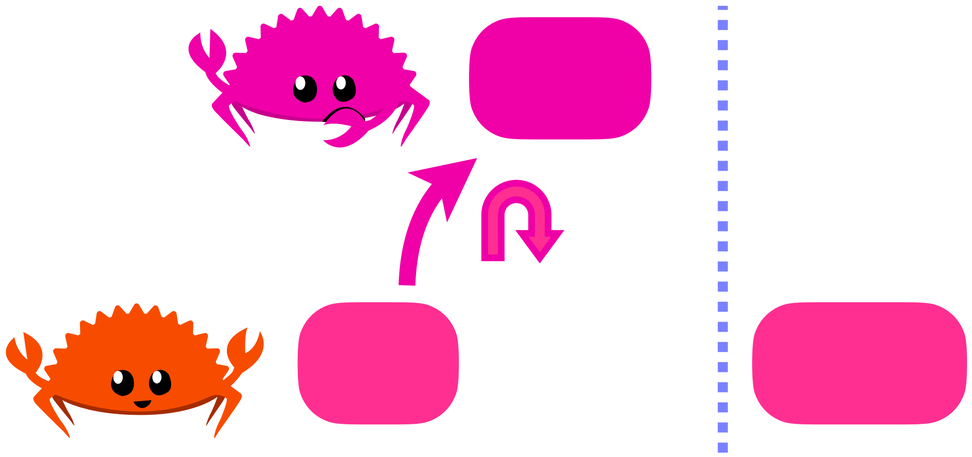
Example

```
example :: Ur (Int, [Int])
example = lin do°
  v ← newVec [0, 1, 2]
  runBO do
    let !(mv, lv) = borrow v
    n ← mut_example mv
    pure $ move $
      (n, freeVec $ reclaim lv)

    Ur (12, [12, 1, 7])
```

```
mut_example :: Mut^α (Vec Int) ⊸ BO^α Int
mut_example mv = do
  let !(mv₀, mv₁) = splitAt mv 1
  (Ur n, mv₁) ← parBO    Can't keep it?
    (do mv₀ ← modifyAt mv₀ 0 (+ 3)
        modifyAt mv₀ 0 (× 4)
        let !(Ur sv) = share mv₀
        copyAt sv 0)
    (modifyAt mv₁ 1 (+ 5))
  let () = consume mv₁; pure n
```

Reborrow

Reborrow



$$\text{reborrowing} :: \text{Mut}^{\alpha}\ a \multimap$$

$$(\forall\, \beta.\ \text{Mut}^{\beta \wedge \alpha}\ a \multimap \text{BO}^{\beta \wedge \alpha'}\ r) \multimap$$

$$\text{BO}^{\alpha'}\ (r, \underline{\text{Mut}^{\alpha}\ a})$$

Lifetime intersection

Example

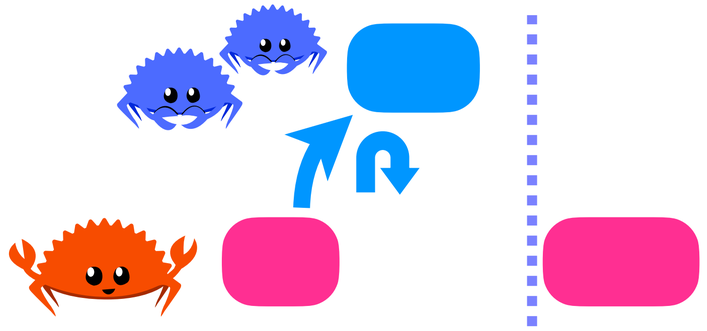
```
example :: Ur (Int, [Int])
example = lin do°
  v ← newVec [0, 1, 2]
  runBO do
    let !(mv, lv) = borrow v
    n ← mut_example mv
    pure $ move $
      (n, freeVec $ reclaim lv)

    Ur (12, [12, 1, 7])
```

```
mut_example :: Mut^α (Vec Int) ⊸ BO^α Int
mut_example mv = do
  ((), mv) ← reborrowing mv λmv →
    let !(mv₀, mv₁) = splitAt mv 1
    consume <$> parBO
      (do mv₀ ← modifyAt mv₀ 0 (+ 3)
          modifyAt mv₀ 0 (× 4))
      (modifyAt mv₁ 1 (+ 5))
  let !(Ur sv) = share mv; copyAt sv 0
```

Temporary Sharing

Temporarily share

$$\begin{aligned}&\text{sharing} :: \text{Mut}^{\alpha}\ a \multimap \\ &\quad (\forall\, \beta.\ \text{Share}^{\beta \wedge \alpha}\ a \to \text{BO}^{\beta \wedge \alpha'}\ r) \multimap \\ &\quad \text{BO}^{\alpha'}\ (r, \text{Mut}^{\alpha}\ a)\end{aligned}$$



```
sharing m k =
  reborrowing m λm →
    let !(Ur s) = share m in k s
```

Read from Mut

$$\begin{aligned}&\text{copyAtMut} :: (\text{Copyable}\ a, \beta \leq \alpha) \Rightarrow \\ &\quad \text{Mut}^{\alpha}\ (\text{Vector}\ a) \to \text{Int} \to \\ &\quad \text{BO}^{\beta}\ (\text{Ur}\ a, \text{Mut}^{\alpha}\ (\text{Vector}\ a))\end{aligned}$$

```
copyAtMut i mv =
  sharing mv $ copyAt i
```

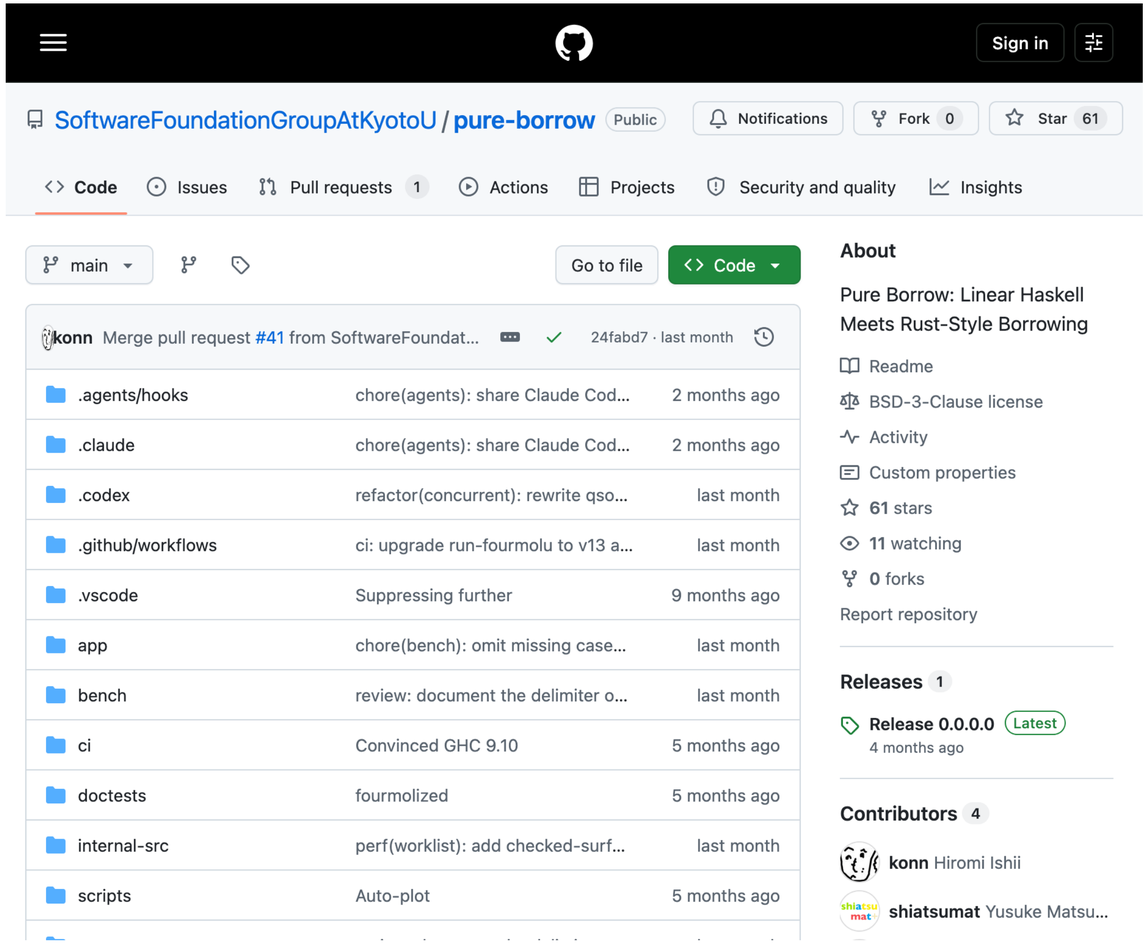
Remark **Internal Implementation**

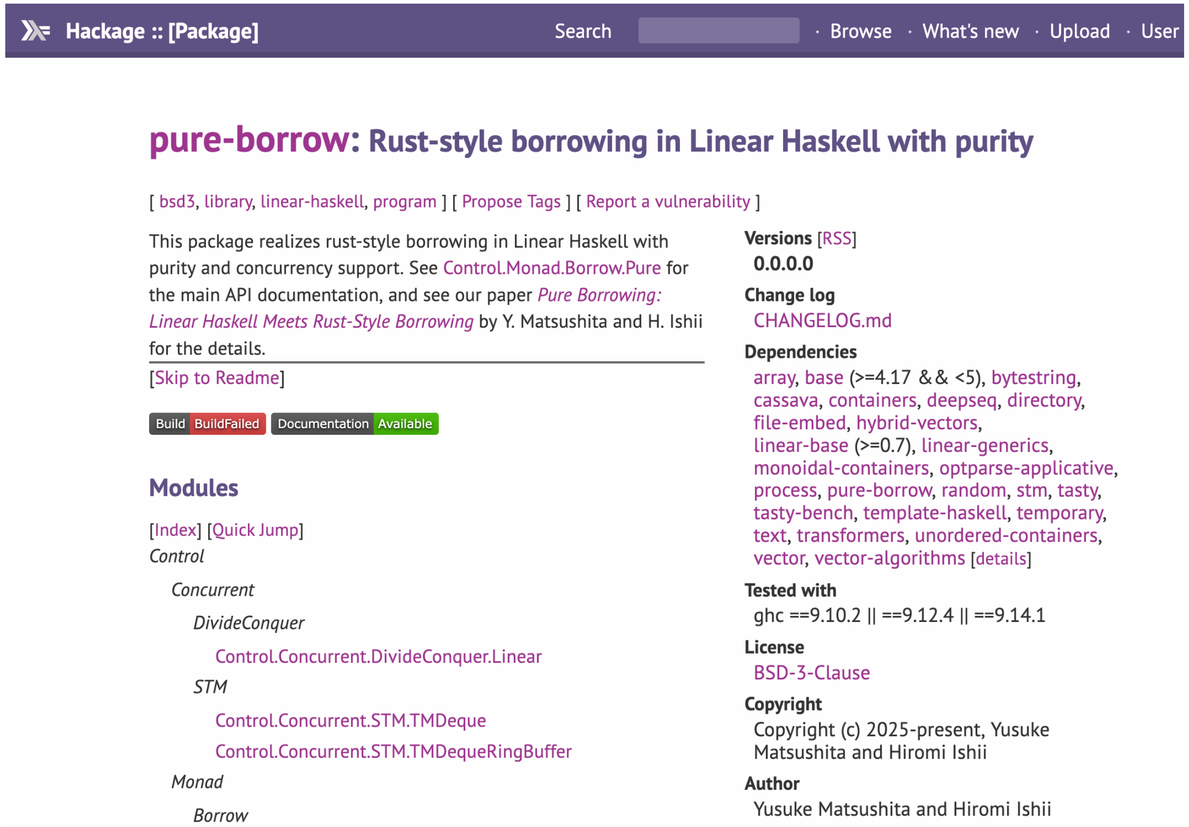
Just unsafe coercions → Efficient

$$\begin{aligned} \text{borrow} &= \text{unsafeCoerce}\ \lambda a \to (a, a) \\ \text{reclaim} &= \text{unsafeCoerce}\ \lambda a \to a \\ \text{share} &= \text{unsafeCoerce}\ \lambda a \to \text{Ur}\ a \end{aligned}$$

Case Study

Pure Borrow is Already There!

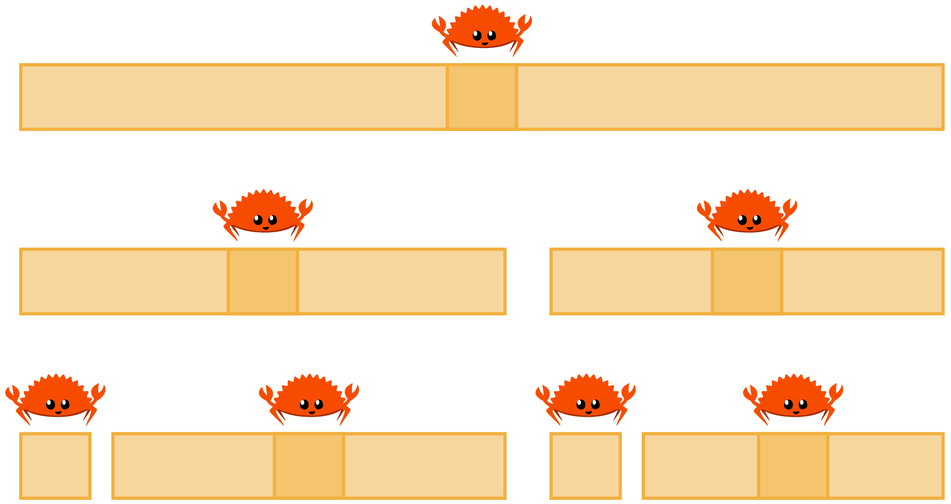




Pure Borrow works on stable GHC

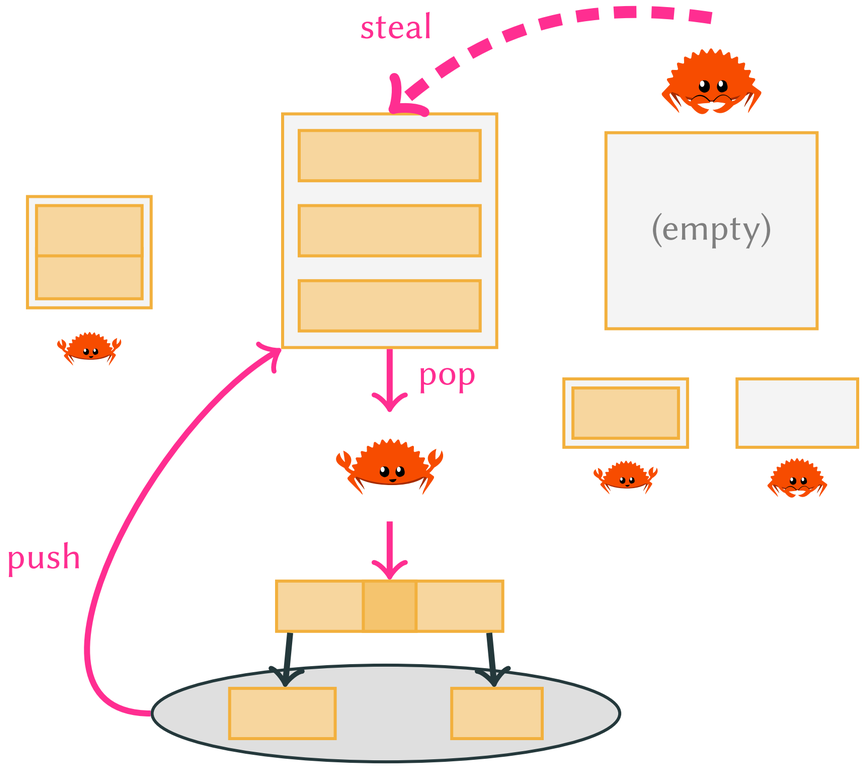
Parallel Quicksort

Pure* + *Mutate* + *Parallel



```
qsort :: Mut^α (Vec Int) ⊸ BO^α ()
qsort v = let (Ur n, v) = size v in
  if n ≤ 1 then pure (consume v) else do
    (Ur a, v) ← copyAt v (n // 2)
    (i, v) ← pivotSwap v a
    let (lo, hi) = splitAt v i
    consume <$> parBO (qsort lo) (qsort hi)
```

Advanced: Work-Stealing Scheduler

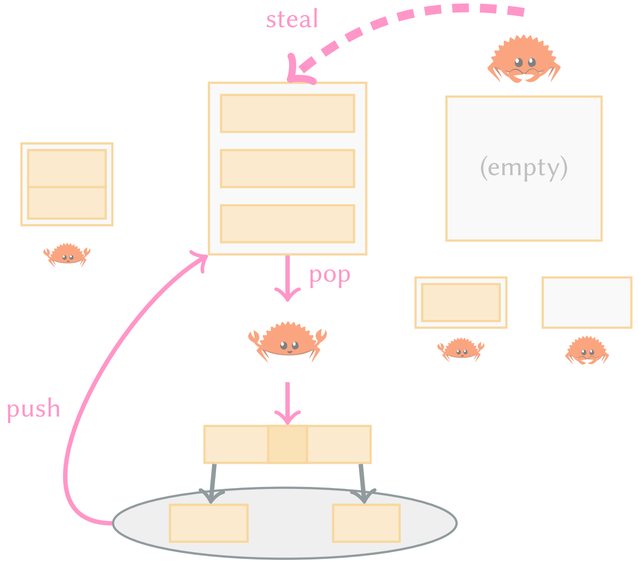


$$\textbf{data}\ \mathrm{DivideConquer}^{\alpha}\ \mathrm{a} = \mathrm{DivideConquer}\ (\forall\,\beta.\ \beta \le \alpha \Rightarrow \mathrm{Mut}^{\beta}\ \mathrm{a} \multimap \mathrm{BO}^{\beta}\ (\mathrm{Res}^{\beta}\ \mathrm{a}))$$

$$\textbf{data}\ \mathrm{Res}^{\beta}\ \mathrm{a} = \mathrm{Done} \mid \mathrm{Cont}\ [\mathrm{Mut}^{\beta}\ \mathrm{a}]$$

$$\mathrm{divideAndConquer} :: \beta \le \alpha \Rightarrow \mathrm{Int} \to \mathrm{DivideConquer}^{\alpha}\ \mathrm{a} \to \mathrm{Mut}^{\alpha}\ \mathrm{a} \multimap \mathrm{BO}^{\beta}\ (\mathrm{Mut}^{\alpha}\ \mathrm{a})$$

Parallel Quicksort by Work Stealing

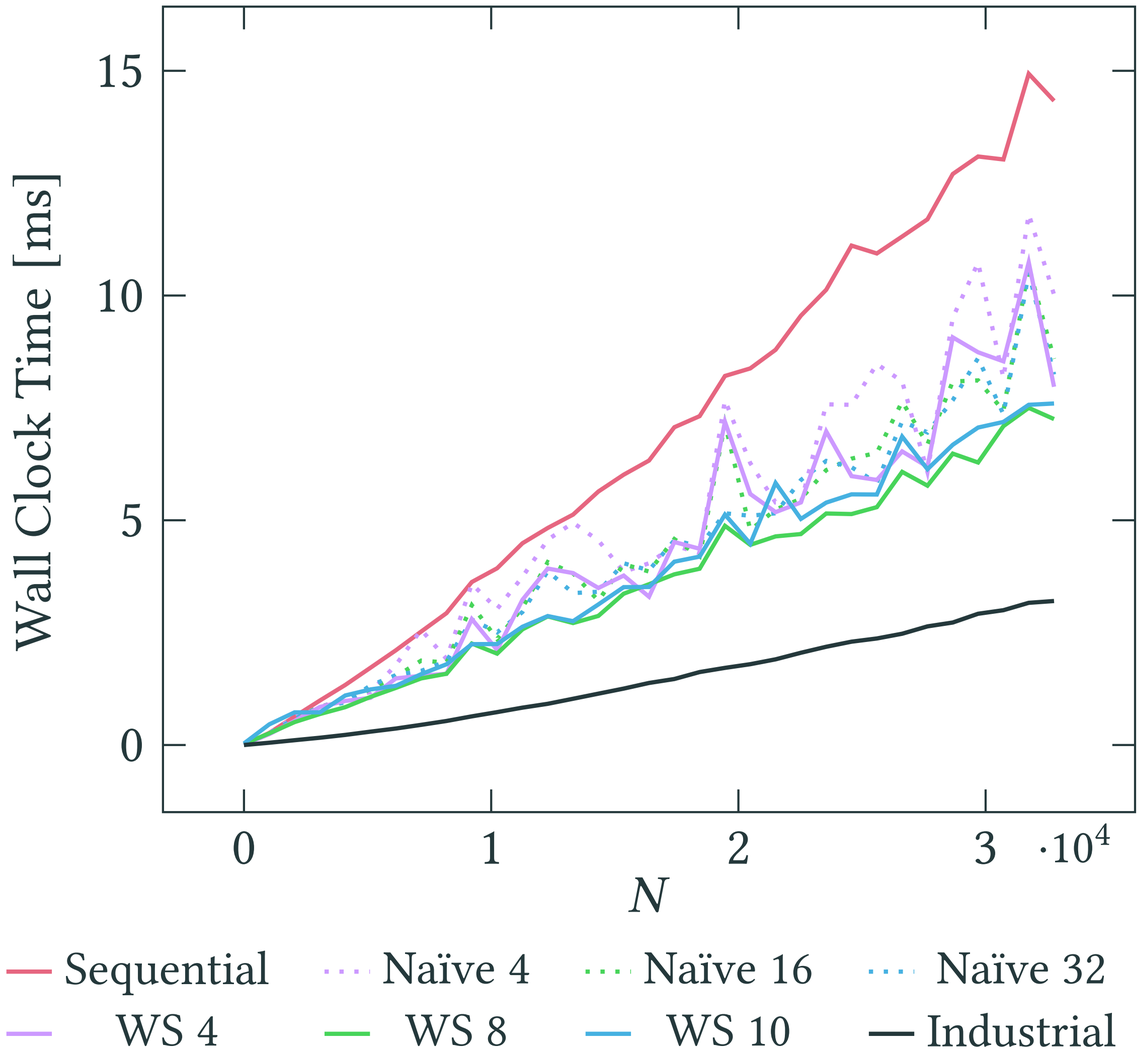



```
data DivideConquer^α a = DivideConquer
        (∀ β. β ≤ α ⇒ Mut^β a ⊸ BO^β (Res^β a))

data Res^β a = Done | Cont [Mut^β a]

divideAndConquer :: β ≤ α ⇒
    Int → DivideConquer^α a →
    Mut^α a ⊸ BO^β (Mut^α a)
```

```
qsortDC :: β ≤ α ⇒ Int →
    Mut^α (Vec Int) ⊸ BO^β (Mut^α (Vector Int))
qsortDC nwork = divideAndConquer nwork $
    DivideConquer λv → case size v of
        (Ur n, v)
            | n ≤ 1 → let () = consume v in pure Done
            | otherwise →
                (Ur a, v) ← copyAtMut v (n // 2)
                (lo, hi) ← pivotSwap v a; pure $ Cont [lo; hi]
```

No thread joining needed → Efficient

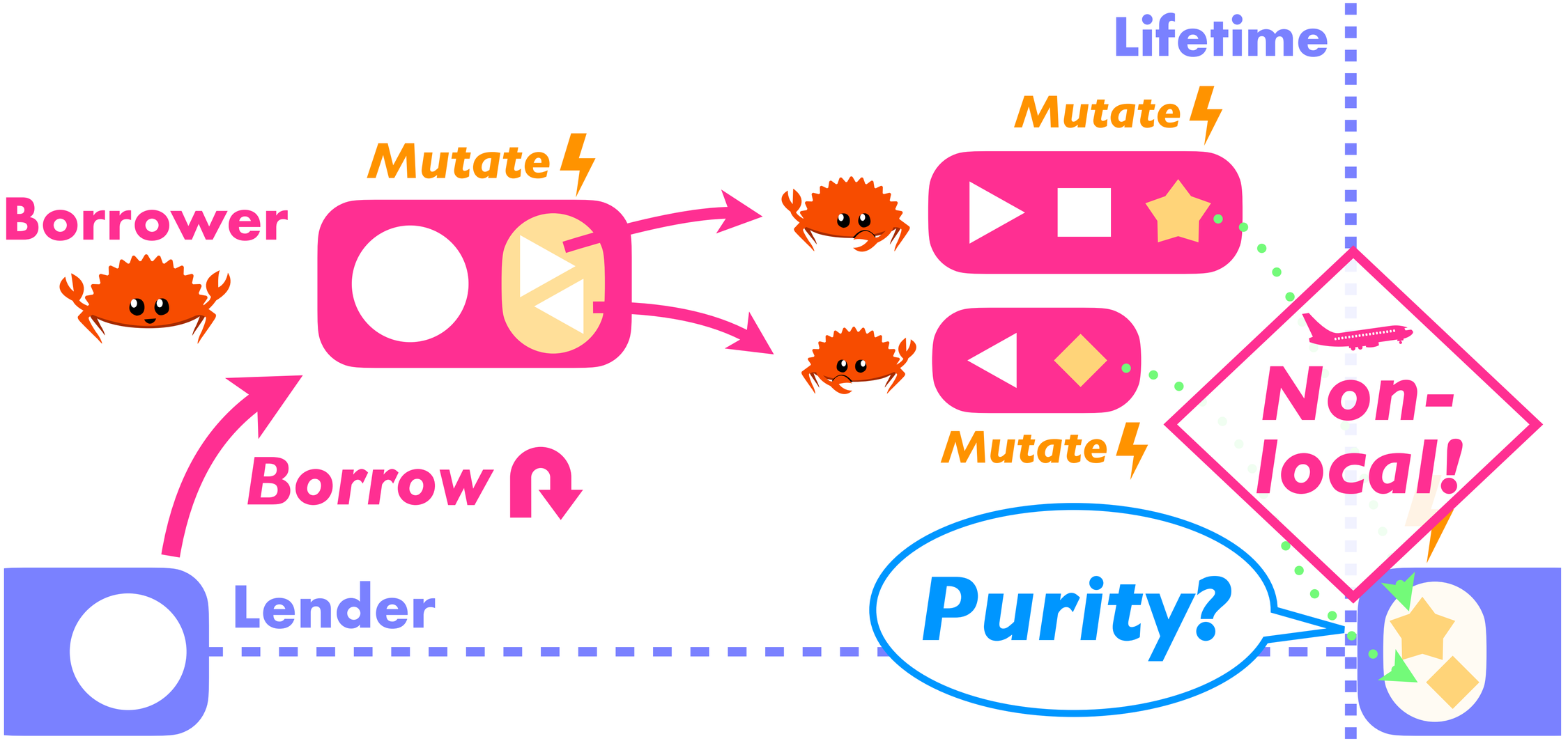
Benchmark Results



Successfully parallelized

Metatheory

Challenge: Purity of Borrowing



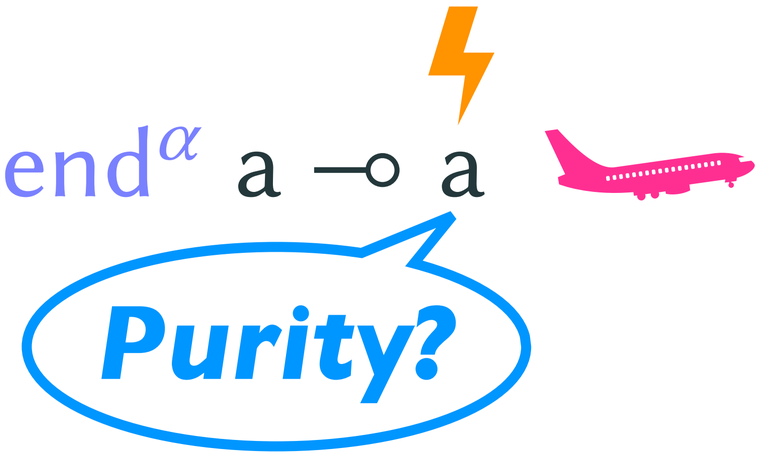
Challenge: Purity of Borrowing

Borrow $\quad \text{borrow} :: \text{Lin} \multimap a \multimap (\text{Mut}^{\alpha}\ a, \text{Lend}^{\alpha}\ a)$

Mutate $\quad \text{modifyAt} :: \text{Mut}^{\alpha}\ (\text{Vec}\ a) \multimap \text{Int} \to (a \multimap a) \multimap \text{BO}^{\alpha}\ (\text{Mut}^{\alpha}\ (\text{Vec}\ a))$

Split $\quad \text{splitAt} :: \text{Mut}^{\alpha}\ (\text{Vec}\ a) \multimap \text{Int} \to (\text{Mut}^{\alpha}\ (\text{Vec}\ a), \text{Mut}^{\alpha}\ (\text{Vec}\ a))$

Reclaim $\quad \text{reclaim} :: \text{End}^{\alpha} \Rightarrow \text{Lend}^{\alpha}\ a \multimap a$



Idea: Histories

- Manage the global history of mutations to borrows for each lifetime
- BO monad: State monad with the history state
- Lifetime end class: Owns frozen history
- reclaim: Restore the updated value from the history

Our Formalization & Metatheory

- ✦ Core calculus + Linear type system for Pure Borrow
- ✦ Two operational semantics
  - ▸ Mutative, with memories & Denotational, based on histories
- ✦ Theorem: Denotational op. sem. is confluent
  - ▸ Full purity: Future work, due to fresh borrow id generation
- ✦ WIP: Mutative & denotational opsems bisimulate?
  - ▸ For well-typed programs & configurations

Reference **Core Calculus Syntax**

Variable $\quad Var \ni x, y, z \qquad$ Data constructor $\quad C$

$$
\begin{aligned}
\text{Operator} \quad op \ ::= \ & (+)\ 2 \mid \cdots \mid (\leq)\ 2 \mid \cdots \\
& \mid \mathsf{par}\ 2 \mid \mathsf{consume}\ 1 \mid \mathsf{move}\ 1 \\
& \mid \mathsf{lin}\ 1 \mid \mathsf{withLin}\ 1 \mid \mathsf{newRef}\ 2 \mid \mathsf{freeRef}\ 1 \\
& \mid \mathsf{newLifetime}\ 2 \mid \mathsf{endLifetime}\ 1 \mid \mathsf{execBO}\ 2 \\
& \mid \mathsf{borrow}\ 2 \mid \mathsf{share}\ 1 \mid \mathsf{copy}\ 1 \mid \mathsf{joinMut}\ 1 \mid \mathsf{reclaim}\ 2
\end{aligned}
$$

$$
\begin{aligned}
\text{Monad constructor} \quad mo \ ::= \ & \mathsf{pure}\ 1 \mid (\gg\!=)\ 2 \mid \mathsf{sexecBO}\ 2 \\
& \mid \mathsf{parBO}\ 2 \mid \mathsf{deref}\ 1 \mid \mathsf{updateRef}\ 2
\end{aligned}
$$

$$
\begin{aligned}
\text{Term} \quad Term \ni t, u \ ::= \ & x \mid \mathbf{let}\ \overline{x = t}\ \mathbf{in}\ u \mid \lambda x \to t \mid t\ u \mid \mathbf{seq}\ x\ t \mid n \\
& \mid C\ \bar{t} \mid \mathbf{case}\ t\ \mathbf{of}\ \{\overline{C\ \bar{x} \to u}\} \mid op\ \bar{t} \mid mo\ \bar{t}
\end{aligned}
$$

Reference **Core Calculus Type Syntax**

Lifetime variable α, β Lifetime id ι

Lifetime $\alpha, \beta ::= \mathrm{al}\,\iota \mid \mathrm{static} \mid \alpha \wedge \beta \mid \alpha$

Multiplicity variable π, μ Multiplicity $\pi, \mu ::= 1 \mid \omega \mid \prod \overline{\pi}$

Type variable X, Y Variable $x ::= \alpha \mid \iota \mid \pi \mid X$

Type constructor F Borrower constructor $B ::= \mathrm{Mut} \mid \mathrm{Share}$

Type $\mathit{Type} \ni T, U ::= \forall x.\, T \mid X \mid T \to_\pi U \mid F\,\bar{T} \mid \mathsf{Int} \mid \mathsf{Lin}$
$\mid \mathsf{Ref}\,T \mid \mathrm{Now}^\alpha \mid \mathrm{End}^\alpha \mid B^\alpha\,T \mid \mathrm{Lend}^\alpha\,T \mid \mathrm{BO}^\alpha\,T$

$T \multimap U \triangleq T \to_1 U$ $T \to U \triangleq T \to_\omega U$

Data type declaration $\mathbf{data}\ F\,\bar{X}\ \mathbf{where}\ \overline{C :: \overline{T \to_\pi F\,\bar{X}}}$

Typing context $\Gamma, \Delta ::= \overline{x ::_\pi T}$

Related Work

Recap **High-Level Comparison**

	Pure 	Mutate	Borrow	Parallel	Leak-free
Pure Only	●	○		●	
ST Monad	●	●	○	○	○
Rust	○	●	●	●	○
Basic Linear Haskell	●	●	○	●	●
Pure Borrow	●	●	●	●	●

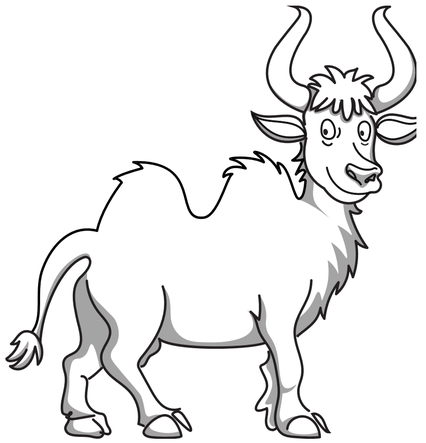
Comparison with Monads

	Pure	Mutate	Borrow	Scope	Parallel	Leak-free
IO Monad *Peyton Jones+ '93*	○	●	○	○	●	●
ST Monad *Launchbury+ '94*	●	●	○	○	○	○
Monadic Regions *Fluet+ '04*	○	●	○	●	●	◐
RIO Monad *Bernardy+ '18*	○	●	○	○	●	●
Pure Borrow	●	●	●	●	●	●

OxCaml

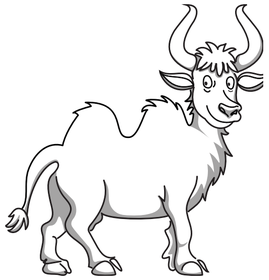
Lorenzen+ ’24, Georges+ ’25

- OCaml + Safe low-level control
  - Ownership, memory-usage, thread safety, …
- Lightweight mode annotation
  - Easy to add to legacy OCaml code
  - Coarse lifetime modes: global vs local
- No strict purity



Linearity vs Uniqueness

Marshall+ '22



Linearity many ≤ once

Uniqueness unique ≤ aliased

Comonad

Monad

Non-linearity Ur

Comonad

Ur a ⊸ a

Ur a ⊸ Ur (Ur a)

Time-delimited aliasing Mut^{α} & Lend^{α}

Monad-like

a ⊸ (Mut^{α} a, Lend^{α} a)

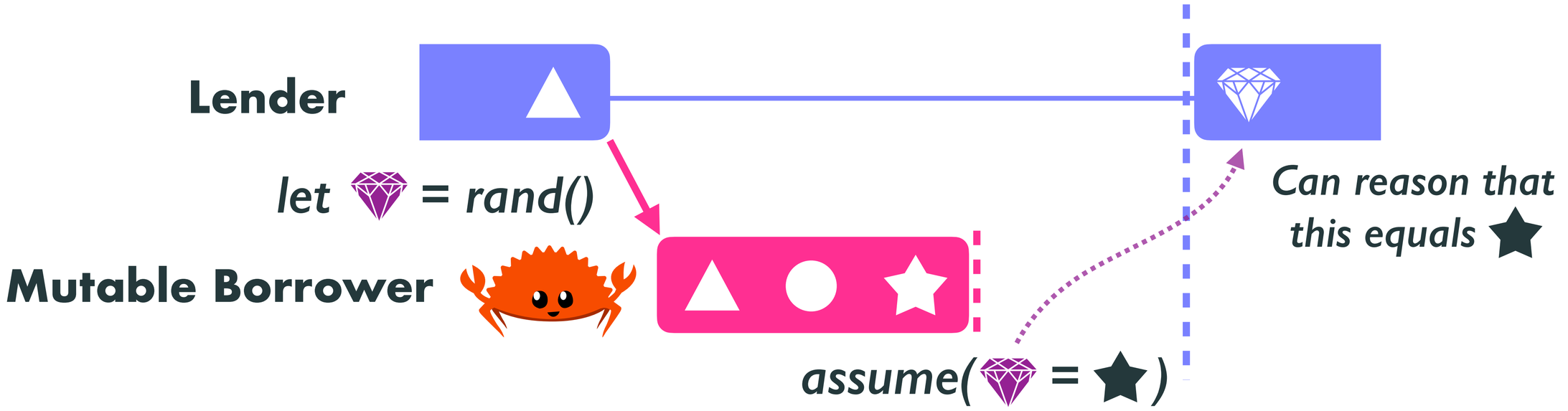
Mut^{α} (Mut^{β} a) ⊸ $\mathrm{Mut}^{\alpha \wedge \beta}$ a

RustHorn

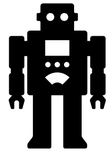
Matsushita+ '20 &'21

- Verify Rust programs as functional code
- Model mutable borrows by prophecy
  - Prophecy [Abadi+ '88]: Fetches info about the future

Final result of the borrower's mutation



Example of RustHorn'' Translation

Rust 

Functional 

```
fn take_max<α>
     (ma: &α mut i32, mb: &α mut i32)
      -> &α mut i32 {
  if *ma >= *mb { ma } else { mb }
}

fn test(mut a: i32, mut b: i32) {
  *take_max(&mut a, &mut b) += 7;
  assert!((a - b).abs() >= 7);
}
```

```
let take_max (a, a') (b, b') =
  if a >= b then
       (assume(b' = b); (a, a'))
  else (assume(a' = a); (b, b'))

let test a b =
  let a' = rand() in let b' = rand() in
  let (r, r') =
    take_max (a, a') (b, b') in
  assume(r' = r + 7);
  assert(abs (a' - b') >= 7);
```

Prophecies vs Histories

Prophecies 

- Concise & local
  - Ideal for **verification**
- Non-determinism
- Precise resolution needed
  - Otherwise **over-approximate**

Histories 

- Rather bulky & global
- Deterministic
- Robust
  - Ideal for the **purity** proof

Conclusion

Summary: Pure Borrow

- **Linear Haskell + Rust-style borrowing**
  - Pure & Mutate w/o ownership passing & Parallelize
  - BO monad for lifetime constraints
- Case study: Parallel quicksort
- Metatheory: Purity by history-based model

Future Work

- **Logical relations** like RustBelt [Jung+ '18] ?
  - Using separation logic [O'Hearn+ '01] such as **Iris** [Jung+ '15] ?
- **Denotational semantics?**
  - Fresh **borrow id** generation, as in ν**-calculus** [Pitts+ '93]
  - **Purity** & **Equational theory**?
  - Prove **soundness**? Using **logical relations**?
- **BO monad transformer?**

Thank you!