Nola: Later-free ghost state for verifying termination in Iris

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Takeshi Tsukada Chiba University

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Shared Mutable State × Termination is Hard

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Landin's knot

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let r = ref (λ_.()) in
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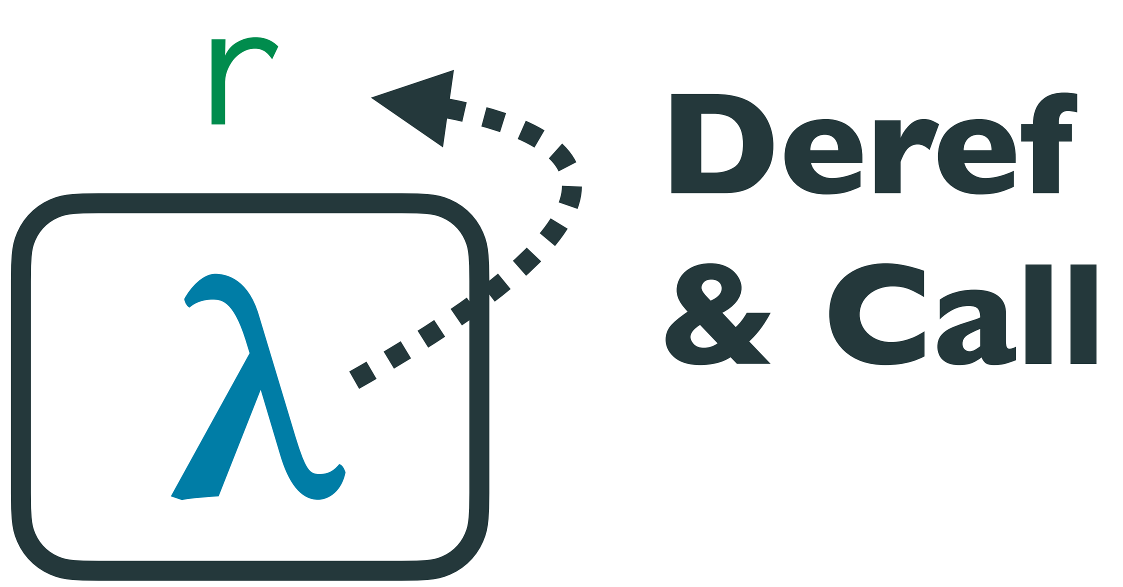
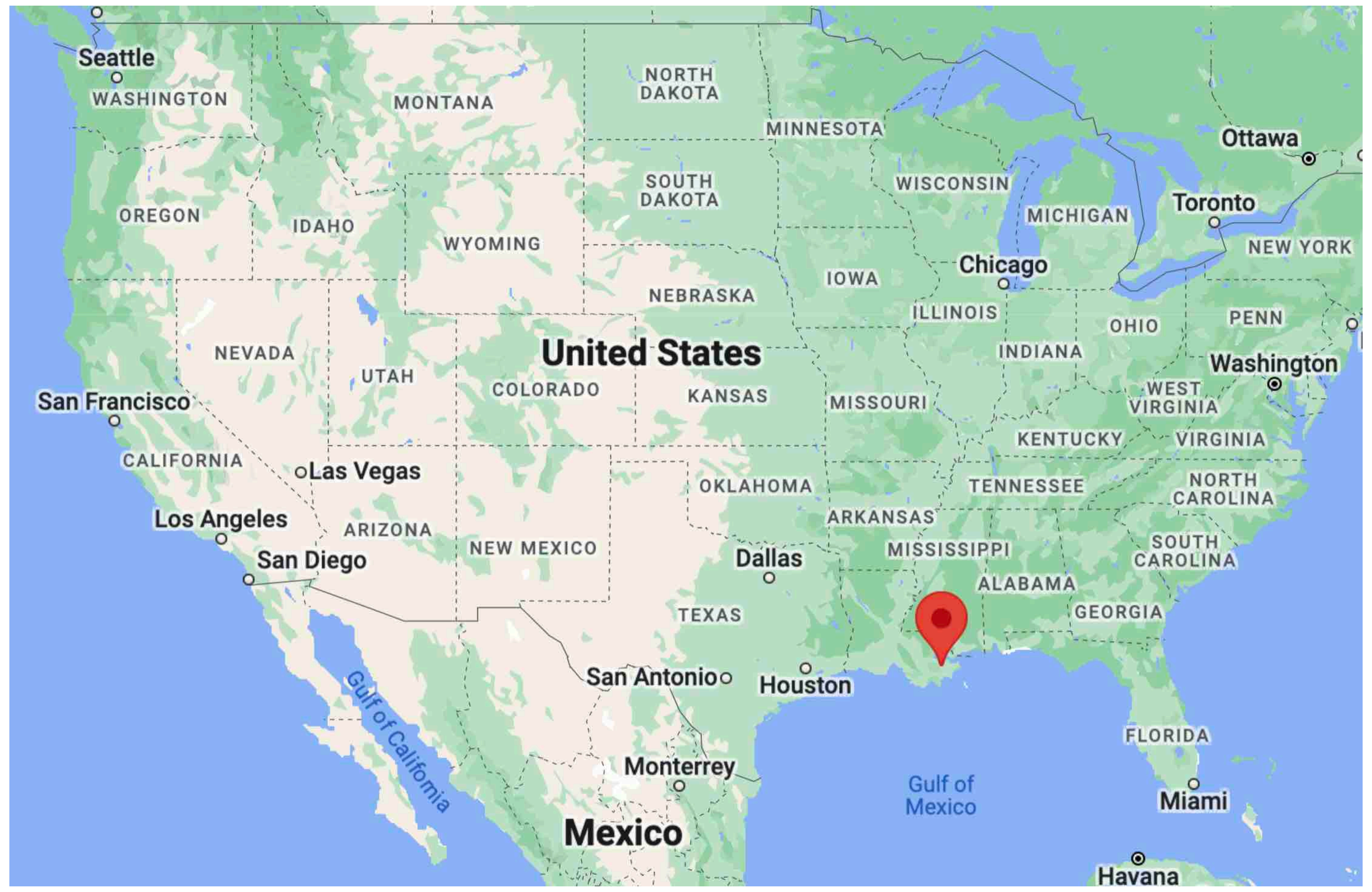


Self-reference

Our work, Nola

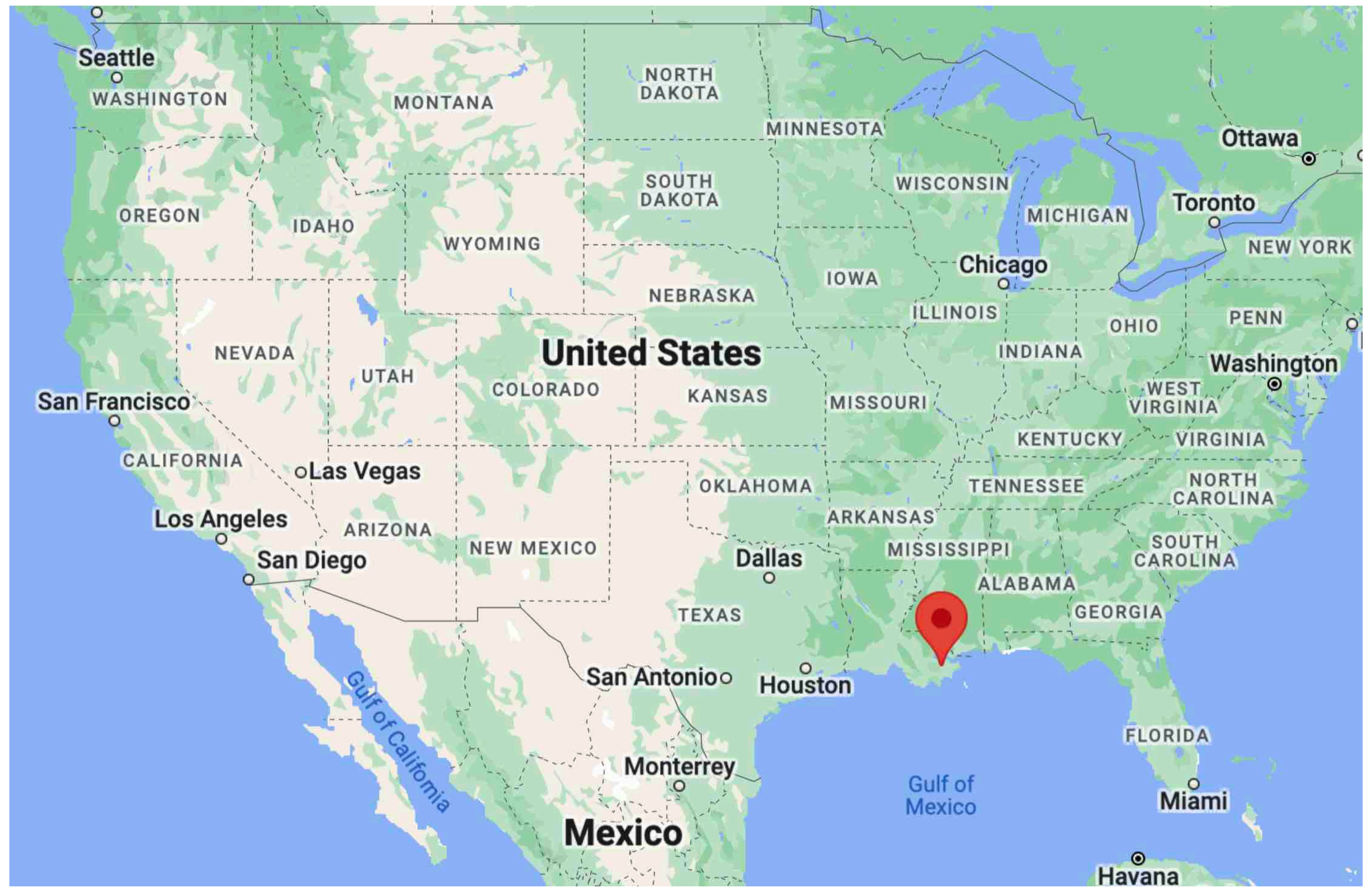
Our work, Nola

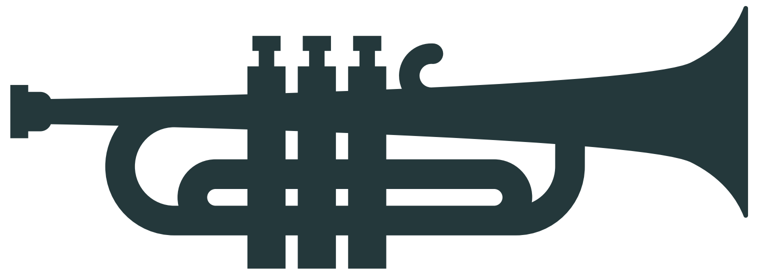


New Orleans, Louisiana

= NOLA

Our work, Nola





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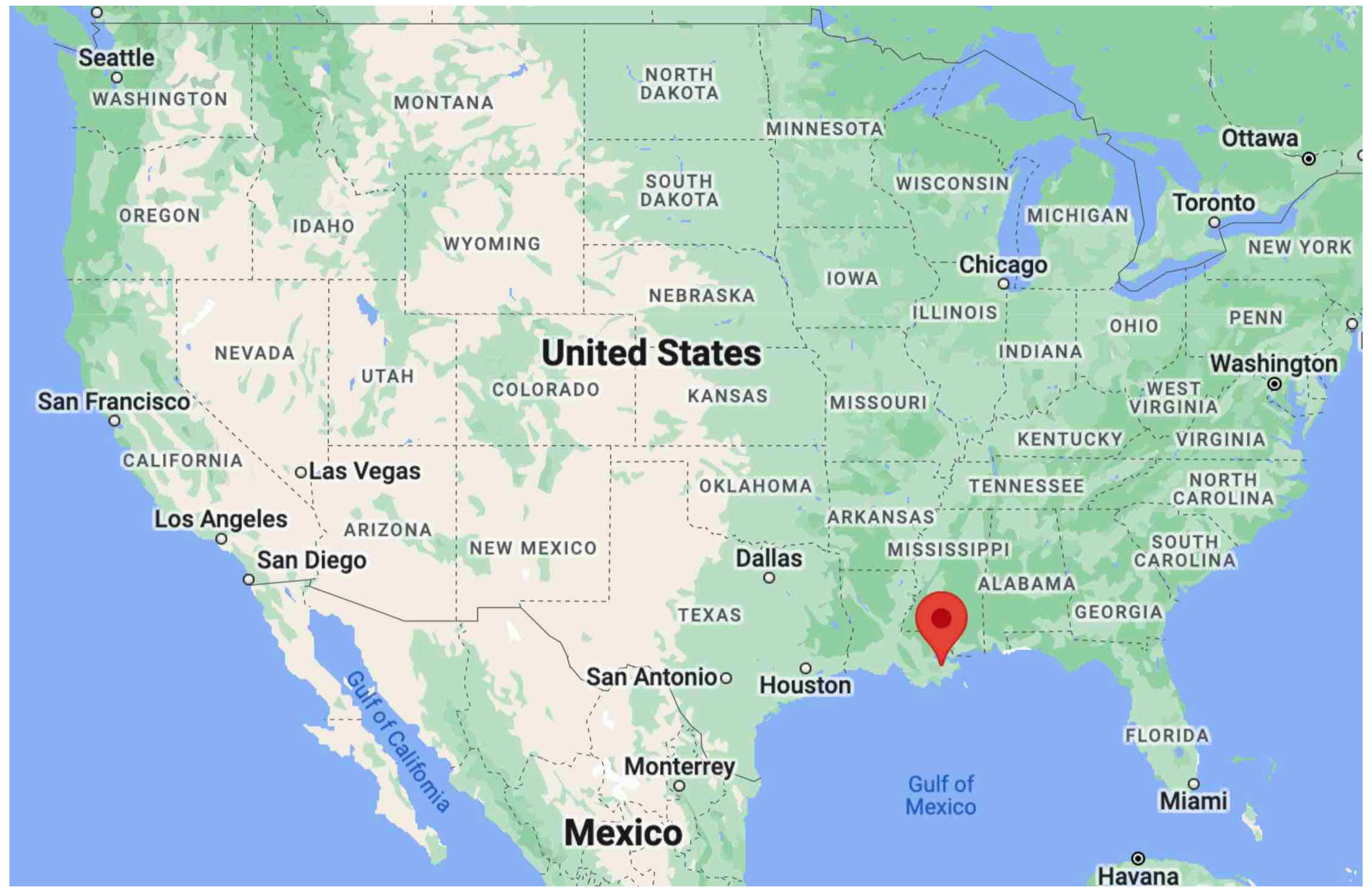

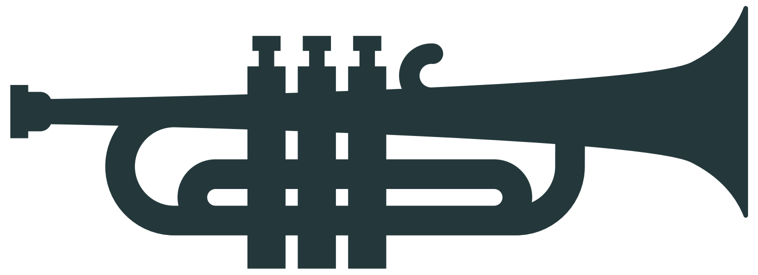

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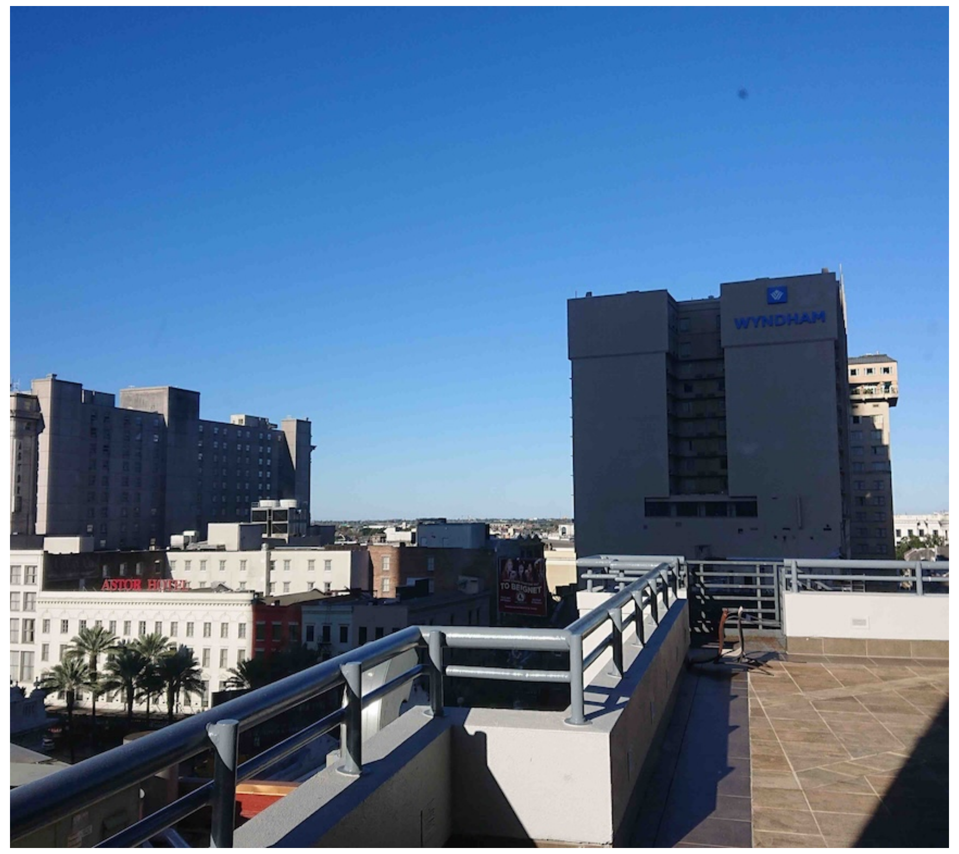

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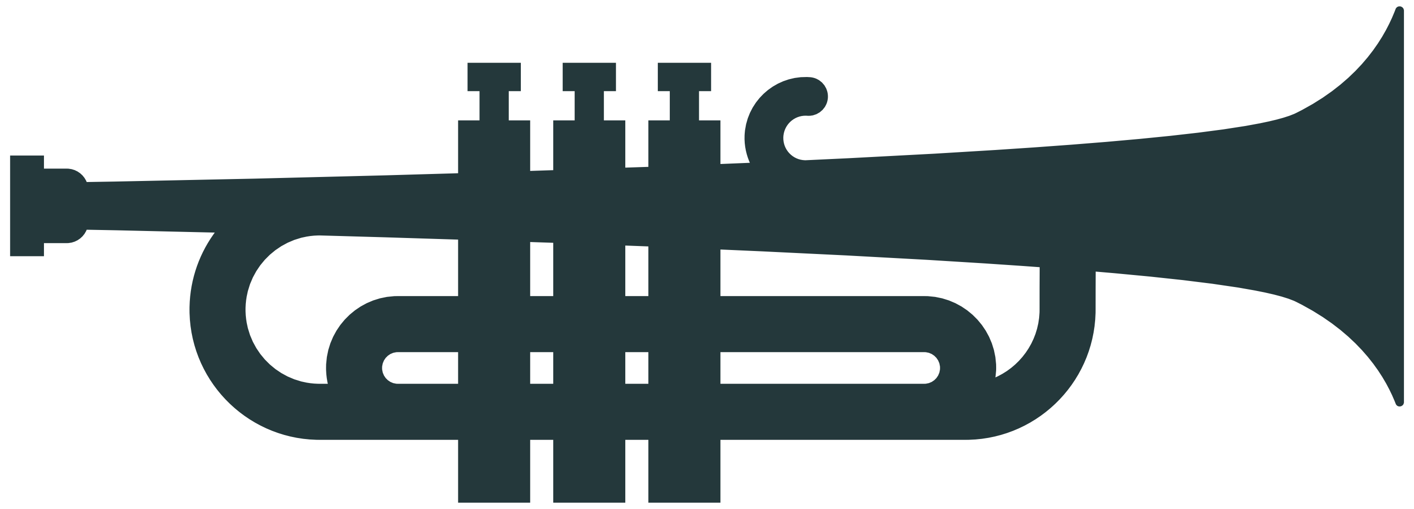

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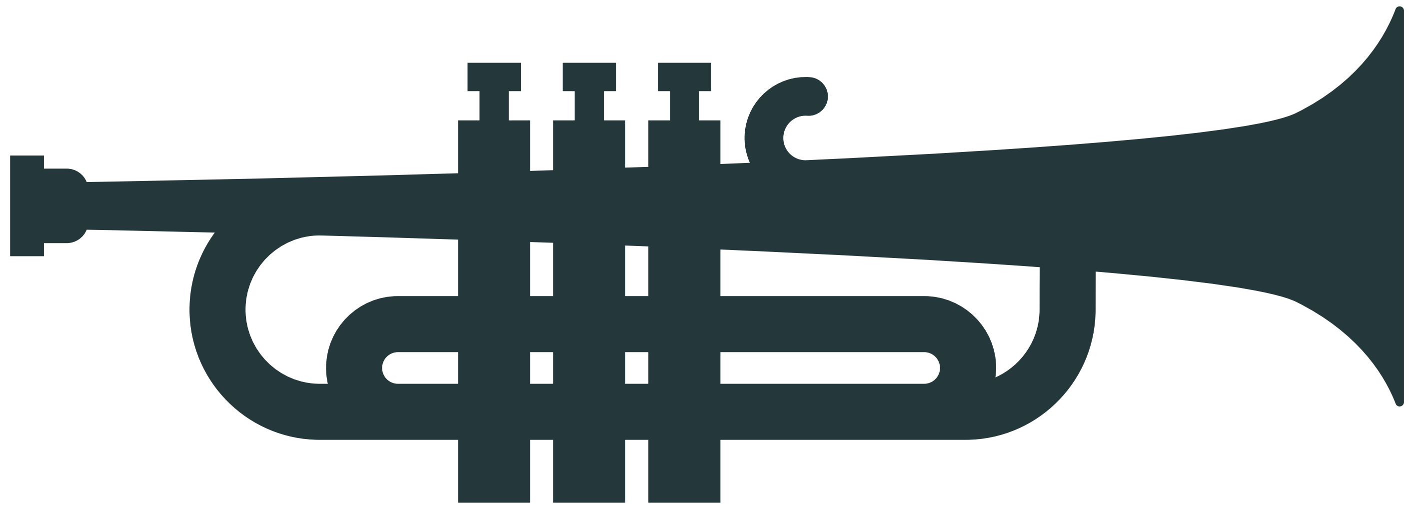
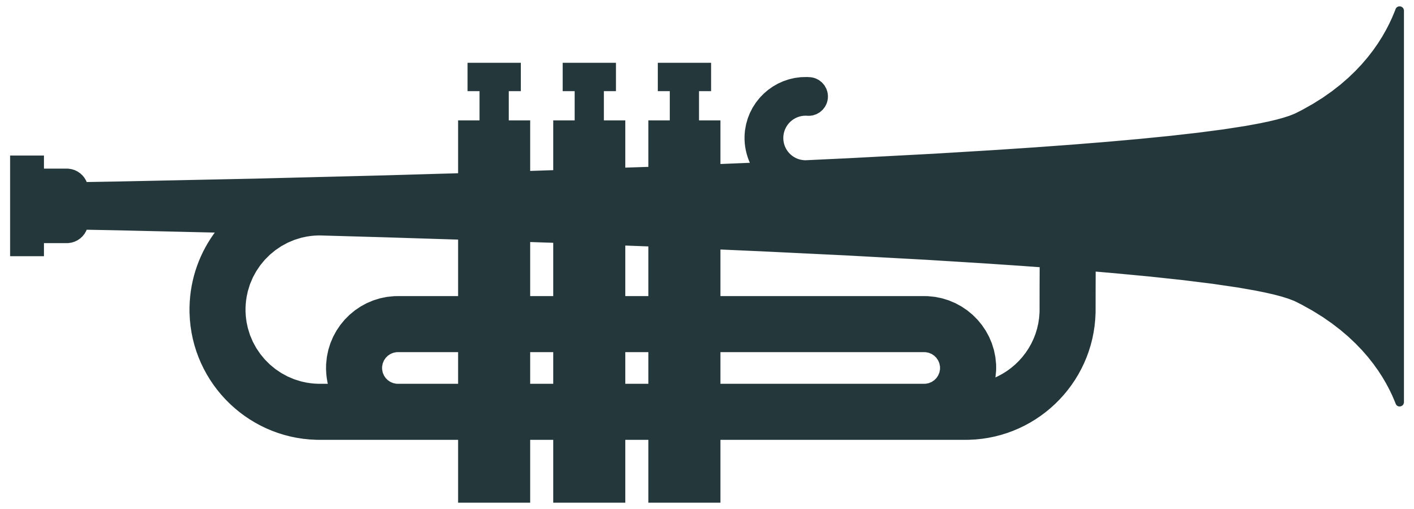


Nola

No later

Our work, Nola



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Helps you finish your work…

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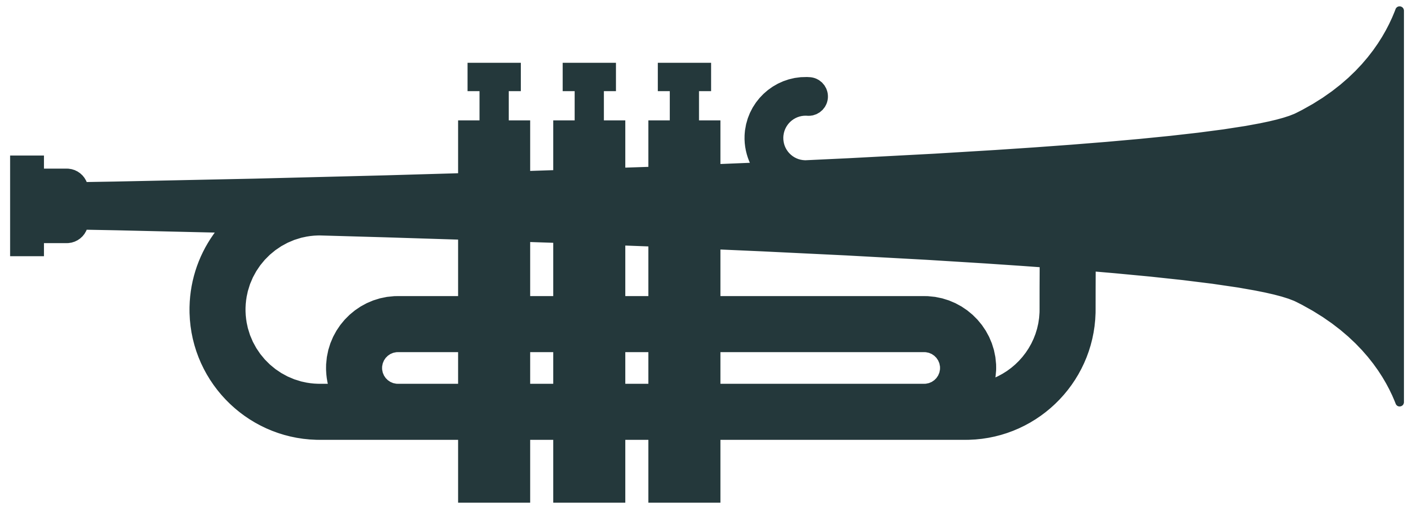

Later modality ▷

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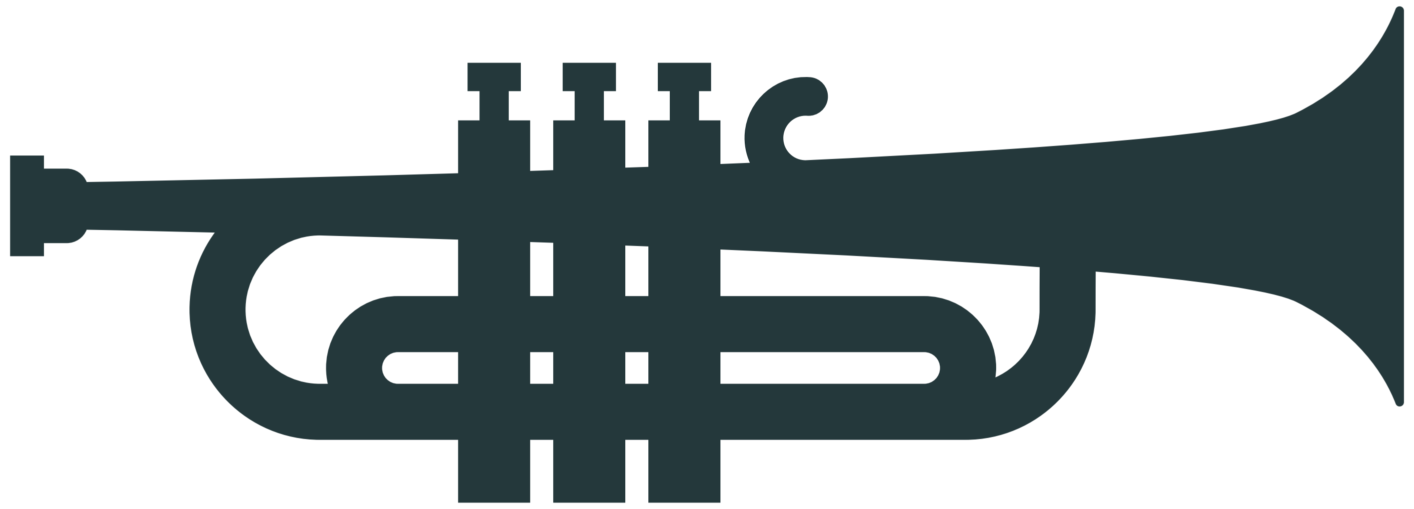

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Iris Separation Logic

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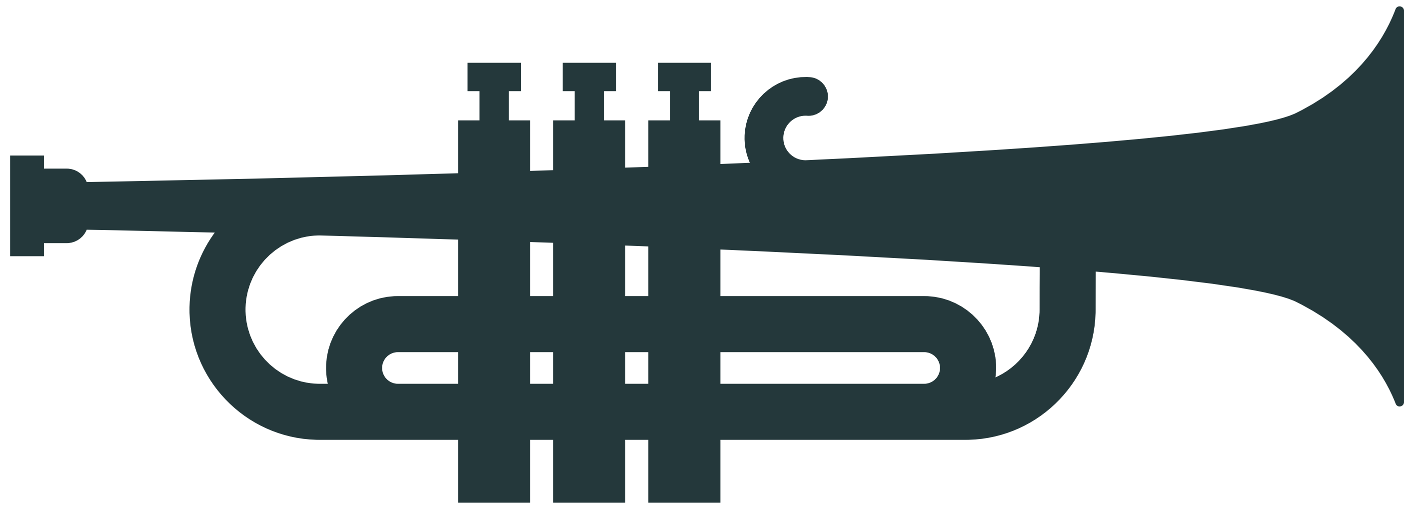

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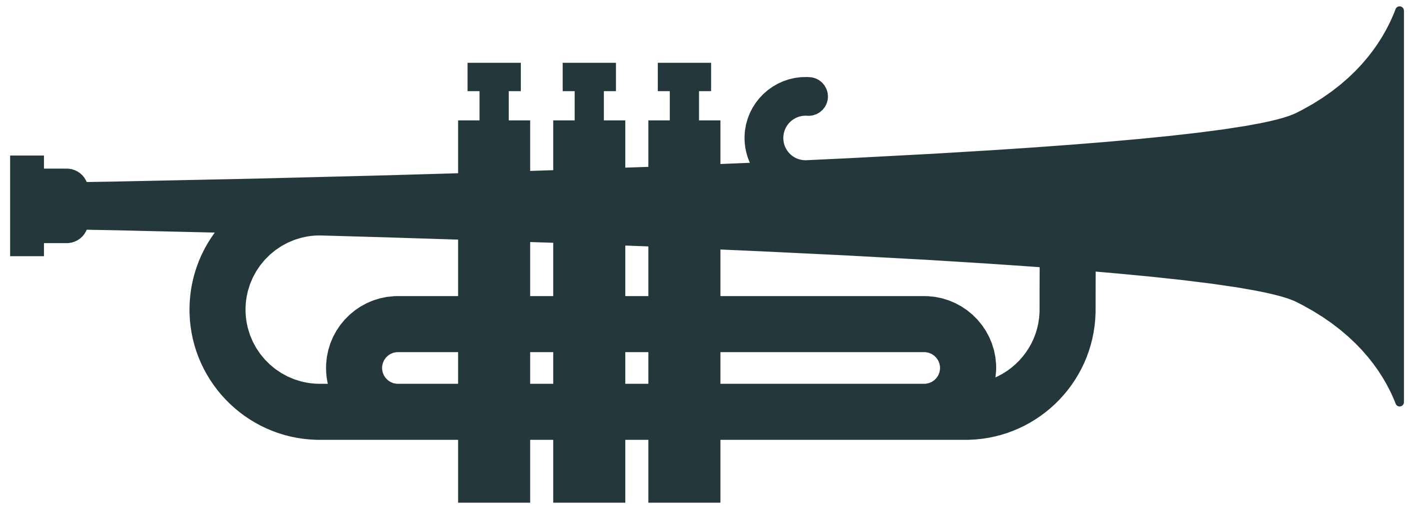

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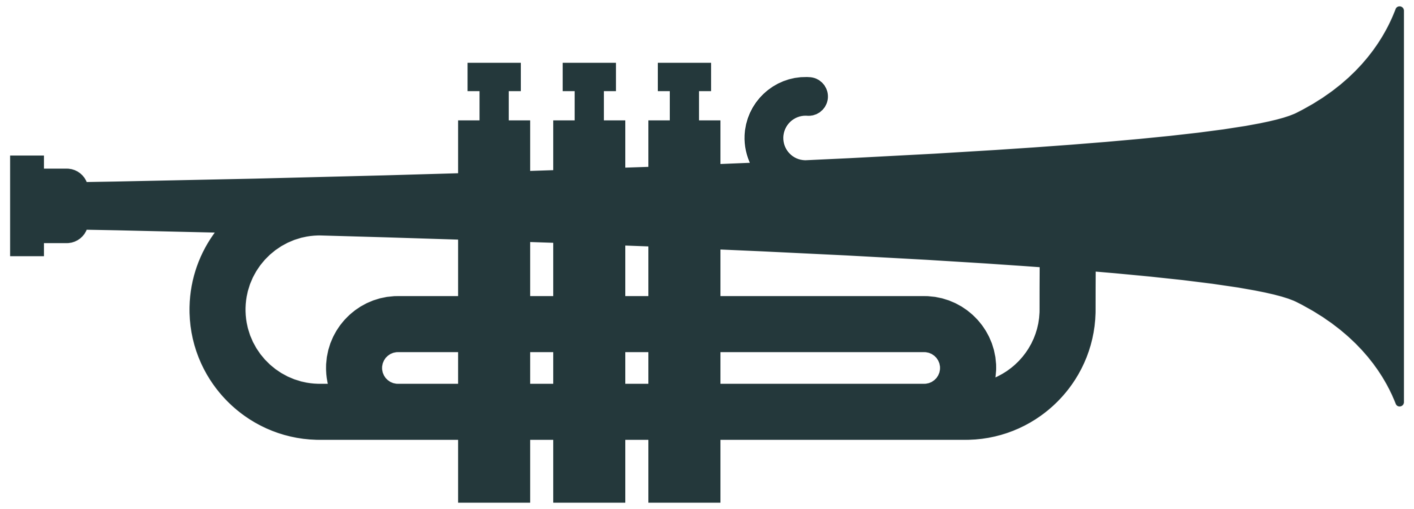

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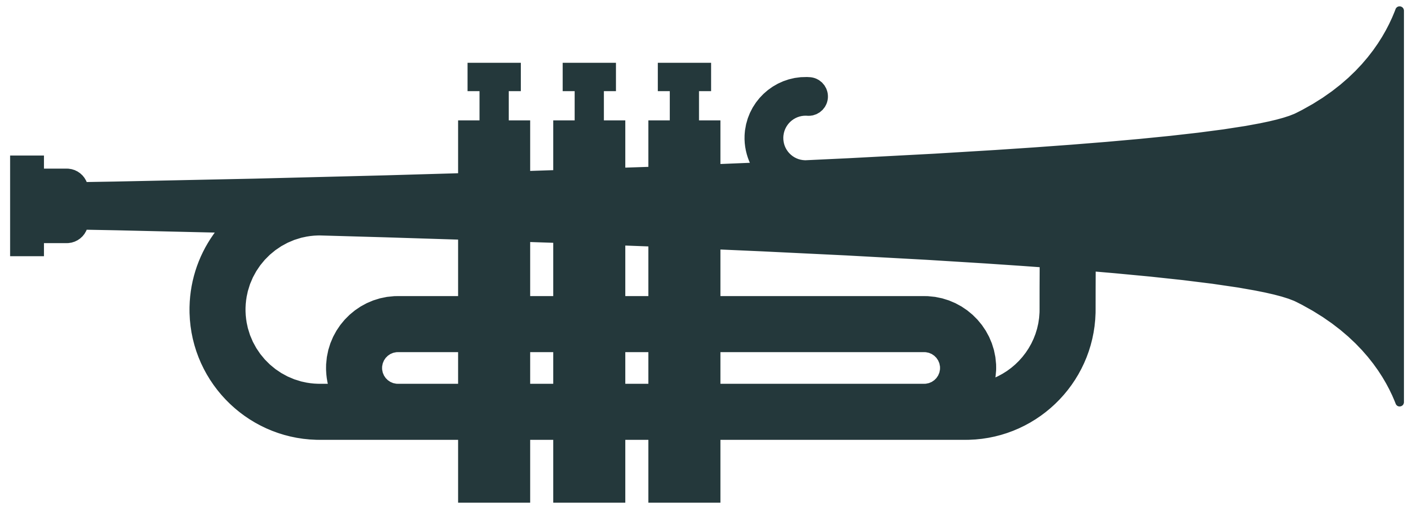

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Invariants & Borrows

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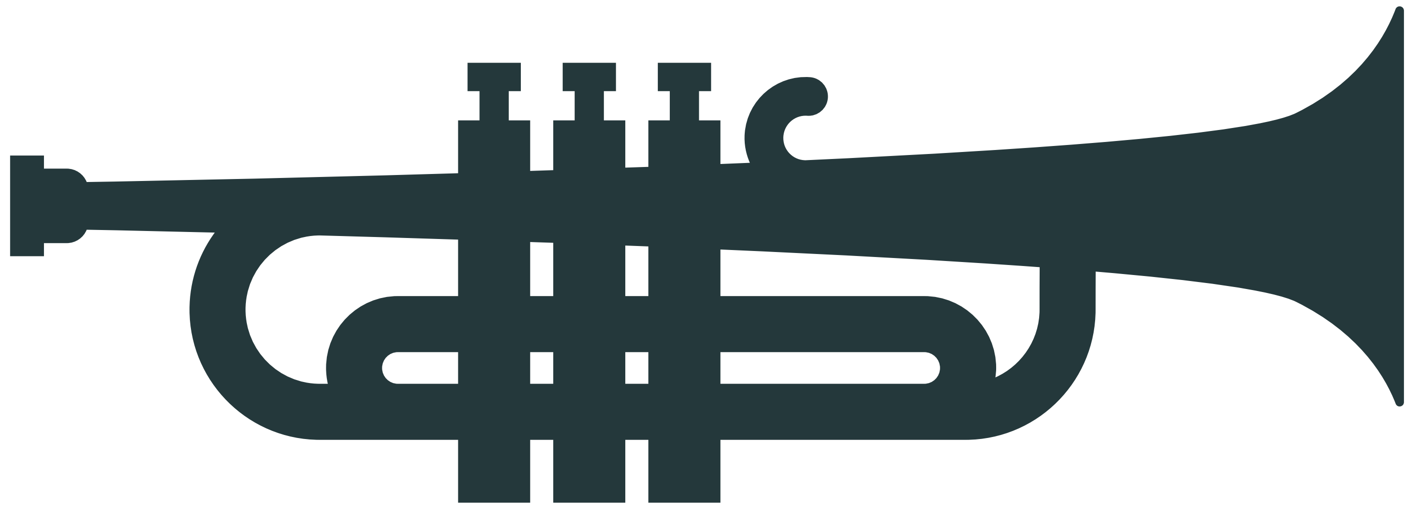

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Separation Logic

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Sound

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Invariants & Borrows

Sound Termination?

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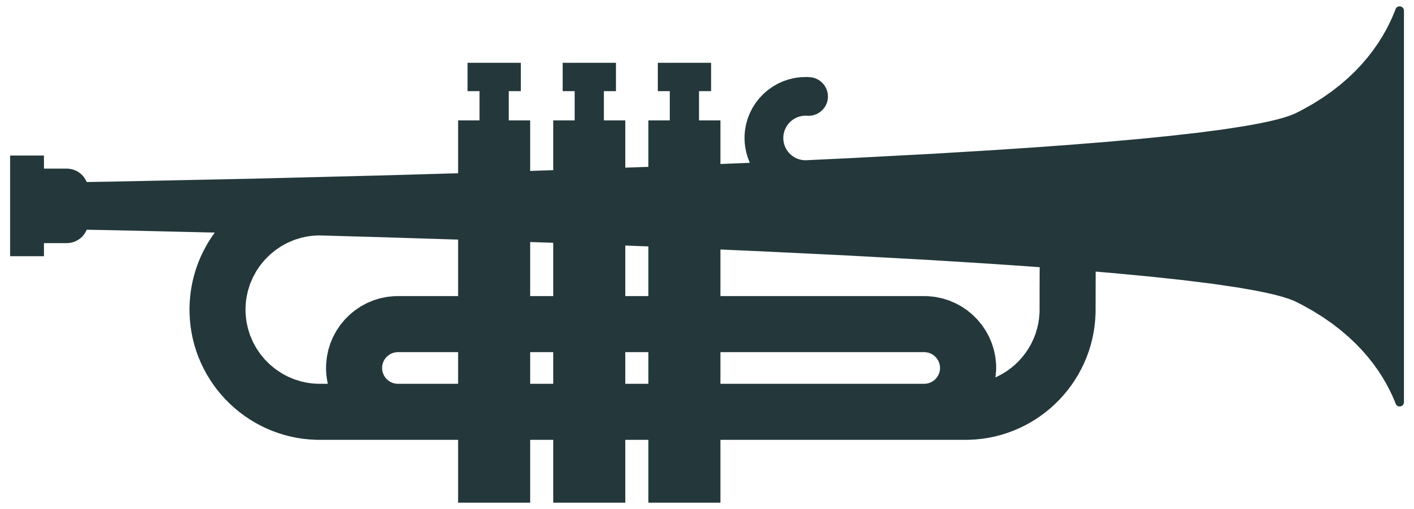

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Invariant $\boxed{P}$ Modern usage established by Iris [Jung+ '15]

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    $\because$ **State mutation** $[\mathtt{r} \mapsto \mathtt{v} * P]\ {*\mathtt{r}} = \mathtt{w}\ [\mathtt{r} \mapsto \mathtt{w} * P]$

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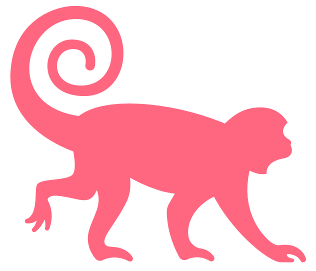

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Iris

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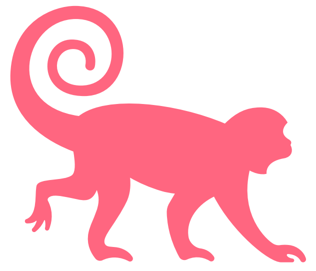

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Blocks access

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Partial $\{\, r : \mathtt{ref}\ T \,\}\ {*r}\ \{\, v.\ v : T \,\}$

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Partial $\left\{\, r : \mathtt{ref}\ T \,\right\} \ast r \left\{\, v.\ v : T \,\right\}$ Step-indexing

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"However, Iris's use of **step-indexing** means that **Iris-**based approaches … do **not** support reasoning about **liveness** properties such as **termination** preservation."

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Use syntax! For **SL assertions** to share

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Syntax gives you **better control** of what can be **shared**, than the **later** modality $\triangleright$, to achieve **soundness**

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$$\frac{[\llbracket P \rrbracket * Q]\ e\ [\llbracket P \rrbracket * R]^{\mathrm{Winv}\,\llbracket\ \rrbracket}}{[\boxed{P} * Q]\ e\ [R]^{\mathrm{Winv}\,\llbracket\ \rrbracket}}$$

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Matsushita & Tsukada PLDI '25

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$P \in \mathit{Fml}$ **Syntactic SL formula**

Our work, Nola

Use syntax! For **SL assertions** to share

Syntax gives you **better control** of what can be **shared**, than the **later** modality $\triangleright$, to achieve **soundness**

$$\frac{[\llbracket P \rrbracket * Q]\ \mathsf{e}\ [\llbracket P \rrbracket * R]^{\mathrm{Winv}\,\llbracket\ \rrbracket}}{[\boxed{P} * Q]\ \mathsf{e}\ [R]^{\mathrm{Winv}\,\llbracket\ \rrbracket}}$$

$P \in \mathit{Fml}$ **Syntactic SL formula**

$\llbracket\ \rrbracket : \mathit{Fml} \to \mathit{iProp}$ **Semantics**

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Proofs can be written **semantically** with iProp in Iris !

Nola's syntax clears the later modality

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$$\mathit{Fml} \ni P, Q \quad ::=_{\nu,\mu} \quad P * Q \mid P \mathrel{-\!\!*} Q \mid P \lor Q \mid \forall_A \Phi \mid \exists_A \Phi$$
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Invariant

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Invariant

$$[\![\ ]\!] : Fml \to iProp$$

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Later-free access! $\left[[\![\mathtt{r} : \mathbf{ref}\ \mathtt{T}]\!] \right] *\mathtt{r} \left[\mathtt{v}.\ [\![\mathtt{v} : \mathtt{T}]\!] \right]$

Verification example: Infinite shared mutable list

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- **No later** remains if we use **Nola**

$$\mathtt{list}\ \Phi\ r \triangleq \boxed{\Phi\ r} * \boxed{\exists s.\ r{+}1 \mapsto s * \mathtt{list}\ \Phi\ s}$$

$$\left[\ [\![\mathtt{list}\ \Phi\ r]\!]\ \right] *(r{+}1) \left[\ s.\ [\![\mathtt{list}\ \Phi\ s]\!]\ \right]^{\mathrm{Winv}\,[\![\ ]\!]}$$

Iris

$$\mathrm{llist}\ \Phi\ r \triangleq \boxed{\triangleright\,\Phi\ r} * \boxed{\triangleright\,\exists s.\ r{+}1 \mapsto s * \mathrm{llist}\ \Phi\ s}$$

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$$\big[\, \mathrm{llist}\ \Phi\ \mathtt{r} \,\big]\ {*}(\mathtt{r}+1)\ \big[\, \mathtt{s}.\ \triangleright\, \mathrm{llist}\ \Phi\ \mathtt{s} \,\big]$$

- **Can verify termination of iteration naturally!**

```
fn iterc(f,c,r) { if *c > 0 {
    f(r); *c = *c − 1;
    iterc(f,c,*(r+1)) } }
```

$$\frac{\forall \mathtt{r}.\ \big[\, \boxed{\Phi\ \mathtt{r}} \,\big]\ \mathtt{f}(\mathtt{r})\ \big[\, \top \,\big]^{\mathrm{Winv}\,[\![\ ]\!]}}{\big[\, [\![\mathtt{list}\ \Phi\ \mathtt{r}]\!] * \mathtt{c} \mapsto n \,\big]\ \mathtt{iterc}(\mathtt{f},\mathtt{c},\mathtt{r})\ \big[\, \mathtt{c} \mapsto 0 \,\big]^{\mathrm{Winv}\,[\![\ ]\!]}}$$

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- Allows **flexible** construction for **extra expressivity**
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  - **Stratification** $[\![\ ]\!]_i : \mathit{Fml}_i \to \mathit{iProp}$ $\quad [\![\, [P]\ \mathsf{e}\ [Q] \,]\!]_1 \triangleq \left[[\![P]\!]_1\right]\ \mathsf{e}\ \left[[\![Q]\!]_1\right]^{\mathrm{Winv}\,[\![\ ]\!]_0}$

To experts: Power of SL formulas

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- **Any semantic** SL props can be stored under **later**
  - Precisely **subsuming** the **existing** later-weakened approach

$$\mathit{Fml} \ni P, Q \quad ::=_{\nu,\mu} \quad P * Q \mid \cdots \mid \boxed{P} \quad (P \in_\nu \mathit{Fml})$$
$$\mid \check{\triangleright} \hat{P} \quad (\hat{P} \in \blacktriangleright \mathit{iProp})$$

$$[\![\check{\triangleright} \hat{P}]\!] \triangleq \check{\triangleright} \hat{P} \qquad \triangleright P \triangleq \check{\triangleright}\, \mathrm{next}\, P \qquad \check{\triangleright}\, \mathrm{next}\, P \triangleq \triangleright P$$

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- The set of SL formulas can even be **extensible**
  - By **parameterizing** over the constructors, just like **iProp's Σ**

To experts: Nola's model just generalizes Iris's

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- **Nola's model for the invariant generalizes Iris's**
  - Fml for $\blacktriangleright \mathit{iProp}$, $[\![\ ]\!] : \mathit{Fml} \to \mathit{iProp}$ for $\check{\triangleright} : \blacktriangleright \mathit{iProp} \to \mathit{iProp}$

$$\textsc{Inv}\ \mathit{Fml} \ \triangleq\ \textsc{Auth}\left(\mathbb{N} \xrightarrow{\text{fin}} \textsc{Ag}\ \mathit{Fml}\right)$$

$$\boxed{P} \ \triangleq\ \exists \iota.\ \left[\circ\, [\iota \leftarrow \mathrm{ag}\ P]\right]^{\gamma_{\textsc{Inv}}}$$

$$\mathrm{Winv}\ [\![\ ]\!] \ \triangleq\ \exists\, I : \mathbb{N} \xrightarrow{\text{fin}} \mathit{Fml}.\ \left[\bullet\, \mathrm{ag}\ I\right]^{\gamma_{\textsc{Inv}}} * \mathop{\huge\ast}_{\iota \in \mathrm{dom}\, I} \left(\left([\![I\ \iota]\!] * [\mathrm{D}]_\iota\right) \vee [\mathrm{E}]_\iota\right)$$

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Rust-style borrows

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Rust's borrow

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let mut l = 0;
let b = &mut l;
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α (lifetime of `let b = &mut l;` and `*b += 7;`)

Reasoning in SL

$\exists n.\ \mathtt{l} \mapsto n \quad \mathtt{l} : \mathtt{int}$

$\triangle^{\alpha}\ (\exists n.\ \mathtt{l} \mapsto n)$

$\mathtt{l} :^{\dagger\alpha} \mathtt{int}$

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$$[\![P]\!] \Rrightarrow^{\mathrm{Wbor}[\![\ ]\!]} \&^\alpha P * \triangle^\alpha P \qquad \dagger\alpha * \triangle^\alpha P \Rrightarrow^{\mathrm{Wbor}[\![\ ]\!]} [\![P]\!]$$

$$\&^\alpha P * [\alpha]_q \Rrightarrow^{\mathrm{Wbor}[\![\ ]\!]} ⅋^\alpha_q P * [\![P]\!] \qquad ⅋^\alpha_q P * [\![P]\!] \Rrightarrow^{\mathrm{Wbor}[\![\ ]\!]} \&^\alpha P * [\alpha]_q$$

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Example `fn iter(f,l) { match l { Nil => (), Cons(a,l′) => { f(a); iter(f,*l′) } } }`

$$\frac{\forall \mathtt{a}.\ \ \mathtt{a} : \&\alpha\ \mathbf{mut}\ \mathtt{T} \vdash \mathtt{f(a)} \dashv \_.\ \rightsquigarrow \lambda\psi, [(a, a')].\ a' = f\ a \to \psi\ []}{\mathtt{l} : \&\alpha\ \mathbf{mut}\ \mathtt{List<T>} \vdash \mathtt{iter(f,l)} \dashv \_.\ \rightsquigarrow \lambda\psi, [(l, l')].\ l' = \mathrm{map}\ f\ l \to \psi\ []}$$

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Semantics!

$$[\![\Gamma \vdash_\alpha \mathtt{e} \dashv \mathtt{r}.\Gamma' \rightsquigarrow \mathit{pre}]\!] \ \triangleq\ \forall \hat{\psi}, t, q.\ \Big[\ \exists \bar{\hat{a}}.\ \big\langle \lambda\pi.\ \mathit{pre}\,(\hat{\psi}\,\pi)\,(\overline{\hat{a}\,\pi})\big\rangle * [\alpha]_q * [t] * [\![\Gamma]\!](\bar{\hat{a}}, t)\ \Big]$$

$$\mathtt{e}\ \Big[\ \lambda\mathtt{r}.\ \exists \bar{\hat{b}}.\ \big\langle \lambda\pi.\ \hat{\psi}\,\pi\,(\overline{\hat{b}\,\pi})\big\rangle * [\alpha]_q * [t] * [\![\Gamma']\!](\bar{\hat{b}}, t)\ \Big]^{\mathrm{Wrh}\,[\![\ ]\!]}$$

Recent application: Lilo Lee+ OOPSLA ’25

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Example

$$\begin{array}{l}\texttt{while}\ (1)\ \{\ \mathcal{Y};\ X \coloneqq 1; \\ \quad \texttt{do}\ \{\ \mathcal{Y};\ a \coloneqq X;\ \}\ \texttt{while}\ (a = 1);\ \mathcal{Y};\ \mathit{print}(a);\ \}\end{array} \;\Bigg\|\; \begin{array}{l}\texttt{while}\ (1)\ \{\ \mathcal{Y};\ X \coloneqq 2; \\ \quad \texttt{do}\ \{\ \mathcal{Y};\ b \coloneqq X;\ \}\ \texttt{while}\ (b = 2);\ \mathcal{Y};\ \mathit{print}(b);\ \}\end{array}$$

refines

$$\texttt{while}\ (1)\ \{\ \mathcal{Y};\ \mathit{print}(2);\ \} \;\Big\|\; \texttt{while}\ (1)\ \{\ \mathcal{Y};\ \mathit{print}(1);\ \}$$

preserving **termination** under scheduler **fairness**

To experts: Magic derivability, our finding

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- **Semantic** alteration of SL formulas for the body
  - **Goal**: Prove **subtyping** on shared mutable refs **semantically**

Goal $$\frac{\mathsf{T} \le \mathsf{U} \qquad \mathsf{U} \le \mathsf{T}}{\texttt{ref}\ \mathsf{T} \le \texttt{ref}\ \mathsf{U}}$$

Need something like $$\frac{[\![P]\!] \Leftrightarrow [\![Q]\!]}{[\![\boxed{P}]\!] \Leftrightarrow [\![\boxed{Q}]\!]}$$

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- **Magic derivability** enables this by a kind of **fixpoint**

$$[\![\boxed{P}]\!]_\delta \triangleq \exists Q \text{ s.t. } \delta(P \Leftrightarrow Q).\ \boxed{Q}$$

$$\mathit{Judg} \ni J ::= P \Leftrightarrow Q$$

$$[\![P \Leftrightarrow Q]\!]^+_\delta \triangleq [\![P]\!]_\delta \Leftrightarrow [\![Q]\!]_\delta$$

$$\dfrac{\forall\, \delta \in \mathit{Deriv}.\ [\![P]\!]_\delta \Leftrightarrow [\![Q]\!]_\delta}{\forall\, \delta \in \mathit{Deriv}.\ [\![\boxed{P}]\!]_\delta \Leftrightarrow [\![\boxed{Q}]\!]_\delta}$$

$$\mathrm{der}\ J \Rightarrow [\![J]\!]^+_{\mathrm{der}} \qquad \mathrm{der} \in \mathit{Deriv}$$

Our work, Nola

Matsushita & Tsukada PLDI ’25

- **Sound later $\triangleright$-free shared mutable state**
  - Refine Iris’s **invariants** $\boxed{\triangleright P}$ & RustBelt’s **borrows** $\&^{\alpha} \triangleright P$
  - Great for **termination** & **liveness** verification
    - Case study: **RustHalt**, RustHornBelt revised for termination
  - **Fully mechanized** in **Rocq** as a **library** of Iris
- **Syntax P for SL formulas to share:** $\boxed{P}$ **&** $\&^{\alpha} P$
  - **Extensible** & **Semantic** SL props under later
  - **Magic derivability** for **semantic** alteration